On the Origins of Zero

Frits Staal

1. Indic Origins?

IT IS WIDELY believed that zero originated in Indic Civilization but the evidence in support of that belief is not only meager; it is almost zero. No place or time, let alone the name of a discoverer or inventor, has ever been suggested. How can we handle such a problem? We must start from the beginning.

Indic Civilization starts with the Indus Civilization which is earlier than the Vedas. Its inscriptions exhibit occurrences and sequences of circles that resemble the numerals that have expressed zeroes in more recent times; to be a little more precise: more than three thousand years later. Other civilizations roughly contemporary with the urban complexes of Mohenjo-daro and Harappa used circles also, but they did not refer to zeroes. On cuneiform tablets from Uruk in Mesopotamia, dating to 3,000 BCE, circles refer to the number "2" (Tropfke 1980: 29). It weakens the suggestion that the Indus circles were expressions of zero. A more serious difficulty is that the language of the Indus inscriptions is not known. It is not even clear that it was a language and its uses are controversial. Some of these topics are discussed in Staal 2008: 7-11, which provides the evidence and further references.

With Vedic mathematics we are on firmer ground. We find not only geometry but integers, a rudimentary decimal system for counting. It did not include negative numbers, but "addition, subtraction and perhaps multiplication of whole numbers" (Hayashi 2003: 360-61). The Rigveda made use of recursion (Divakaran, *forthcoming*). It did, moreover, distinguish between cardinal and ordinal numbers (Renou: 1964: 92; Staal 2007: 589-590; 2008: 272-273). In all these cases we are dealing with numbers, not with numerals: the Vedas are an Oral Tradition since there was no writing on the subcontinent prior to the Buddhist Emperor Aśoka who reigned from 268 to 231 BCE. But "zero" did not only lack a symbol. There was no term for it in the oral tradition. The word *kha*, which Indian mathematicians used later to denote zero, occurs in Vedic only in the senses of "hole", "opening", "vacancy" or "space".

Counting boards based upon the decimal system took another step but the Indic evidence is of later date and the empty spaces are zeroes of a kind, not symbolic expressions. Even today, the Indian *paṇḍita* or traditional scholar uses neither

counting boards nor books: he carries his knowledge in his head. Writing had its commercial uses, manuscripts go back to the beginnings of the Common Era, but it began to be used widely only after the invention of Hinduism in the early nineteenth century (Staal 2008a).

Fortunately, it is not the end of our story since those who look for the origins of something, even if it is zero, must look beyond the domain where it is customarily located, even if it is absent. The remaining parts of my essay attempt to do so. Part 2 will discuss two pioneering investigations. Parts 3 and 4 will take us beyond the history of mathematics. Part 3 will pay attention to linguistics and Part 4 to the Vedic theory of ritual, not included in modern classifications or curricula though regarded during the period of middle Vedic as an exact science. Having gone that far we must recall, that "exact" and "science" are often no more than labels and that all names of disciplines are due to us, not to the universe to which we belong.

2. Khmer and the Buddhist Madhyamaka

Two original contributions, undigested legacies from the twentieth century, have to be taken into account if we wish to understand how zero may or may not have been discovered. The first is due to Joseph Needham, the famous scholar and scientist who published, together with his collaborators, the many volumes of *Science and Civilization in China*. The second is due to David Ruegg, a brilliant Sanskrit scholar whose early work dealt with the Sanskrit grammarians but whose chief contributions since have been to Buddhist Studies, primarily as expressed in Sanskrit and Tibetan sources.

Needham presented his ideas in the third volume of his series. It is entitled "Mathematics and the Sciences of the Heavens and the Earth" and was first published in 1959. It is important to understand what Needham tried to do for he has been criticized and misunderstood like other pioneers. He did not examine Chinese sciences from a simple "evolutionist" concept of history, as if they were "more or less clumsy attempts to express modern scientific ideas," a notion that Pingree wisely rejects in another context (Pingree 2003: 45). But neither did Needham present Chinese sciences from "the Chinese point of view," whatever that is, as Seyyed Hossein Nasr (1968: 21) tried to do with respect to the Islamic Sciences. Needham's perspective is different and he has expressed it in unambiguous terms: "to write the history of science we have to take modern science as our yardstick—that is the only thing we can do—but modern science will change and the end is not yet" (Needham 1976: xxxi with further discussion in Staal 2006: 91-97).

D. J. de Solla Price, a historian of science at MIT, described Needham's work as follows: "In my estimation, the essential contribution made thus far by the six volumes of *Science and Civilization in China* lies in the systematization and presentation in English translation or summary of the substantive content of the

otherwise ill-digested bulk of Chinese scientific and technical literature. Here we have the raw material on which generations of later scholarship can be founded. Here at last we have some map to tell us where to look, and some indication at least of what we shall find" (1971: 17-18).

Needham's volumes deal with much more than the Chinese sciences. They abound in references to Indian, Mesopotamian and European disciplines. The discussion on the origin of zero begins to meander in Volume 3 on page 9. Needham knows, of course, that zero is widely believed to have originated in Indic Civilization, he is familiar not only with the Chinese but also with Indic counting boards and then takes an unexpected turn: he zeroes in on early South East Asian inscriptions. The thesis that emerges will not surprise us but is not formulated in a few simple sentences or in a single paragraph. We get the idea when we combine three separate sentences that occur on pages 10-12 and that I quote here because they are Needham's own words:

- "The usual view is that the circular symbol for zero derived directly from India, where it first appears on the Bhojadeva inscriptions at Gwalior dated +870."
- "While the first epigraphic evidence for the zero in India is, as has just been mentioned, of the late +9th century, it has been discovered about two centuries earlier in Indo-China and other parts of south-east Asia. This fact may be of much significance."
- "It would seem, indeed, that the finding of the first appearance of the zero in dated inscriptions on the borderline of the Indian and Chinese culture-areas can hardly be a coincidence."

I shall not discuss the first sentence though more recent discoveries of Indian inscriptions have pushed the dates further back. It neither affects Needham's thesis, nor the origins of zero as we shall see. The second sentence is based upon an early article published in 1931 by another celebrated scholar, George Coedès, the French historian of South East Asia and especially of the Khmer Empire of what are now Kampuchea and parts of Thailand. He did not only write the classic account of the 'Indianization' of Indochina and Indonesia that was first published in French, often reprinted since 1964 and translated into English as *The Indianized States of Southeast Asia*. He also published eight volumes of Cambodian inscriptions, and wrote much else that inspired thousands of more recent publications, all of them supplementing, improving and updating his work, just as de la Solla Price had predicted of Needham's series. As for the third of Needham's three sentences, its significance depends on the significance of the second.

Needham's idea is based upon two inscriptions discussed in Coedès article. One comes from Cambodia, the other from Indonesia. Both show the zero, one in the form of a dot and the other in the shape of a small closed curve that may be

a circle. Both are dated 683 CE, almost two centuries earlier than the alleged first Indic inscription of 870.

Needham does not comment on the puzzling circle, if it is a circle, which comes from the small Indonesian island of Banka, but he stumbles on both inscriptions and then makes his fatal lapse. He assumes throughout that their language is Southeast Asian but does not say which language it is. Is he thinking of Khmer or Old Javanese? The simple truth is that the language of the inscriptions is expressed in a Southeast Asian script but is none other than Sanskrit.

This was obvious to Coedès as every careful reader of his article can see. He writes on page 325: "au Cambodge, les premières inscriptions *sanskrites* datées font usage des mots symboliques"; "au Champa, les deux plus anciennes inscriptions *sanskrites* datées . . .en langue *sanskrite*"; "les inscriptions *sanskrites* du Champa"; "à Java, la plus ancienne inscription *sanskrite* datée fait usage des mots symboliques" (all italics mine). These quotes show that "the first appearance of the zero in dated inscriptions on the borderline of the Indian and Chinese culture-areas" does not only fail to be a coincidence. They demonstrate that the content of these inscriptions is Indic.

I now come to David Ruegg who wrote a brief but substantial article partly concerned with our problem. It is entitled "Mathematical and Linguistic Models in Indian Thought: The Case of Zero and *Śunyatā*" and was published in 1978. *Śunyatā* refers, as is well known, to the Buddhist concept of 'emptiness.' It is a characteristic feature of the Madhyamaka school and was foreshadowed by a certain Bhadanta Vasumitra who may have lived at the end of the first or beginning of the second century CE. Its context is the theory of *dharmas*, which do not refer to the Buddhist *dharma* or 'doctrine,' but to 'elements or factors, each of which is considered to bear its own specific characteristic that determines it' as Ruegg explains the expression of the *Abhidharmakośa: 'svalakṣaṇadhāraṇād dharmaḥ'*.

I shall not further some readers' possible annoyance with Vasumitra's Sanskrit but here is Ruegg's translation: "A *dharma* evolving in the [three] times is stated to be other according to the different states it enters, [the change in question being then] due to otherness of state (*avasthāntarataḥ*) but not of substance". The words and phrases within square brackets are due to Ruegg, who explains the example that Vasumitra adds as follows: "like a marker or counter (*vartikā*) in reckoning which in the unit position has the value of a unit, in the hundred's position that of a hundred, and in the thousand's position that of a thousand"—a straightforward expression of the use of zero as a place-marker in the decimal system.

Ruegg adds that the same idea is sometimes expressed by the term *gulikā*, "ball" or "bead" which, like the counting boards to which they belong, should remind us of the fact that mathematicians are not always concerned with what modern readers think of almost exclusively, viz. *writing*. Needham is familiar with this usage of *śunya* because he compares it to the empty spaces on Chinese counterboards in a long footnote on his pages 11-12.

Ruegg discusses two terms for zero: *kha*, which we have already met, and *bindu*, which means "dot". He draws attention to the *Vāsavadattā*, a literary work of the sixth century by Subandhu which uses *śunyabindu* to denote the symbol for zero. He refers next to Pingree's work on the *Yavanajātaka* of Sphujidhvaja, then about to be published. That text was composed in 149/150 CE and used the term *bindu* in "the earliest reference known to the decimal place-value system with a symbol for zero in India" (Pingree 1978, II: 406, I: 494).

The next topic Ruegg considers is the history of the term *śunya*. He starts with the Rigveda which employs *śuna* for "lack, absence, emptiness." (One of the earliest examples is "lack of sons" in RV 7.1.11.) Later Vedic has *śunya* in the meaning "hollow, deserted". After providing more information on *śunyatā*, he concludes cautiously that we cannot trace connections between the Buddhist "emptiness" and the mathematical concept of zero.

Ruegg then turns to early linguistics and Pāṇini's lopa. We shall look at its apparent invisibility in the next section. Ruegg notes its occurrence in modern linguistics but then diverges from our topic in grammatical and philosophical directions that involve *śunyatā* without throwing light on the zero. His cautious peregrinations have inspired my own meanderings. The reader should note what has not been shown and remember Pingree's statement: the earliest written reference to the decimal place-value system with a symbol for zero in India is dated to 149/150 CE. We have to look not only beyond writing but further.

3. Zeroes in Sanskrit Grammar

Almost all Indic mathematicians wrote in Sanskrit, the classical language of science that unites the subcontinent (Staal 1995). Malayalam is among the famous exceptions (Divakaran 2007). Persian and English became more common in premodern and modern times but Sanskrit continued in mathematics and Jyotiḥśāstra or astronomy-cum-astrology (see, e.g., Minkowski 2002, 2008). Many of the classics of Indian mathematics were composed in concise and sometimes elegant Sanskrit verse. Here is young Āryabhaṭa on the subdivisions of time:

"A solar year is a year of men. Thirty of them make an ancestral year. Multiplied by twelve is explained as a year for the gods"(*ravivarṣaṃ mānuṣyaṃ tad api triṃśadguṇaṃ bhavati pitṛyam| pitṛyaṃ dvādaśaguṇitaṃ divyaṃ varṣaṃ samuddiṣṭaṃ*: 3:7) Another couple of lines condenses the full sine table in one couplet (1:12). And who does not know the penultimate verse: "From the ocean of true and false knowledge I have, through the boat of my own knowledge, rescued with the grace of the deity the precious sunken jewel of true knowledge" (*sadasajjñānasamudrāt samuddhṛtaṃ devatāprasādena | sajjñānottamaratnaṃ mayā nimagnaṃ svamatināvā*)

How did these mathematicians know Sanskrit? It could not have been their first or native language. Many were not brahmans (Āryabhaṭa was not or else

his name would have been Āryabhaṭṭa[1]); and neither need all brahmans know Sanskrit. *Ārya*, moreover, does not refer to "the three twice-born classes" as it probably did in the *Manusmṛti*, "The Law Code of Manu", 2.207 (Olivelle 2004: 242). The distinction between *ārya* and *anārya* has not been a racial or ethnic distinction. Madhav Deshpande has shown that it expresses claims to moral, social and spiritual status, tending toward exclusion in so-called 'Hindu' legal texts and epics, but inclusion and transformation among Jainas and Buddhists (Deshpande 1999).

The mathematicians who wrote in Sanskrit, then, might have come from anywhere and their native language may be anyone's guess but they must have studied Sanskrit grammar. It is unlikely that they studied books. In India, no *paṇḍita* or traditional scholar does. An aspiring savant may have been taught by his father or must have had a teacher, who had his own guru, etc. in the oral succession of *guruparamparā*, "the lineage of teachers". All that knowledge must ultimately have come from one of the many existing and surviving Sanskrit grammatical works. Two questions arise: which one did it come from and what did it say?

Like other scholars, many mathematicians are likely to have studied the earliest and most famous Sanskrit grammar: that of Pāṇiṇi of the fifth/fourth century BCE, or its later adaptations such as the *Kāśikā* of the seventh century CE or the Pāṇiniyan grammar of Bhaṭṭoji Dīkṣita of the seventeenth. Buddhists had their own grammars due to famous masters such as Candragomin of the fifth century CE, just as the Jainas had great grammarians from Devanandin (fifth c. CE) to Hemachandra (twelfth) and beyond (for more information on the Sanskrit grammarians see Staal 1972). All these works were inspired, directly or indirectly, by the Pāṇinian tradition. And all of them possessed not one but many zeroes.

What is zero in grammar or linguistics? Pāṇini had a technical term for it: *lopa*. He defined it as "something that does not appear"(*adarśanaṃ lopaḥ* 1.1.60). It is not a rare term in his grammar. Its "non-appearance" (*adarśanaṁ*) does not prevent it from occurring in forty-five out of four thousand rules if I counted them correctly as they were enumerated by Böhtlingk in his edition (1887, with many reprints: II: 271*). The actual number is higher since I have not taken account of Böhtlingk's uses of the expression *fgg* which indicates "and following".

Professor P.P. Divakaran, who commented on an earlier draft of this article, was intrigued by the definition of *lopa* as *adarśanaṃ* because Pāṇini certainly lived before Aśoka who presumably introduced writing in the third century BCE and: "I should think that a sound which is absent would be characterised by Pāṇini as unheard or unsounded rather than as unseen".

[1]The philologist's concept of *lectio difficilior*, "the more difficult reading," should be extended so as to be applicable to the oral transmission of compositions: the more difficult and unexpected of two readings or orally transmitted forms, viz., Āryabhaṭa, must be preferred to the normal, expected form Āryabhaṭṭa, since it is unlikely that it is based upon an error.

This is an apt observation but there is more to say. Some interpreters have indeed translated Pāṇini's *adarśanaṁ* as 'unseen' since the verbal root *dṛś-* certainly means "to see". I translated it differently as "something that does not appear" like others have done because that same root is widely used in the much wider meaning of seeing with one's mind. It includes perception, observation, appearance, knowing, etc., and is a common meaning in Sanskrit and similarly in other languages. In Indian philosophy, the six traditional systems are called *darśana* but their epistemology is not confined to seeing with one's eyes. In English we say: "I see what you mean". In later Sanskrit, *na dṛśyate* means: "it does not appear (that such-and-such is the case)". In English the same ambiguity applies to appear itself: "there appeared a large bird on the roof" suggests that the bird was actually seen but in "there appears to be much confusion about the PM of Thailand stepping down", nothing may have been perceived by eyes or eyes only; it is a topic that people are talking about.

According to Renou's dictionary of grammatical terms in Sanskrit (Renou 1957 under *adarśana*, *lopa* and *lup-*), *adarśana* means *amuissement*, a technical term in French which expresses that a phoneme is dropped in pronunciation. Renou refers to Kātyāyana, grammarian who commented on Pāṇini and lived some two or three centuries later, probably during the reign of Aśoka (a fuller but somewhat opaque discussion occurs in Cardona: 1976, 1980, pages 267-273). That date could support an interpretation that refers to writing, but Kātyāyana himself does not see that way. Whether it was written or not, his statement is startling: *adarśana* means not seen, not heard, not pronounced, not perceived, absence or disappearance. It calls for a closer analysis and the entire subject seems to stand in need of a new and thorough revision but I think that, in the present context, we may safely conclude that *dṛś-* does not only mean "to see with one's eyes", that *adarśanam* does not only mean "unseen" and that "something that does not appear" is the best translation—for the time being.

Mathematicians studying Sanskrit in order to be able to compose works with all-India appeal, could not have missed the numerous zeroes that occur in Sanskrit grammars. Modern readers are able to understand them provided they know something of the morphology of words in Sanskrit. It is found in other Indo-European languages such as English; and in others. The examples that follow below under (1)-(3) occurred in three papers by Pandit: 1962, undated and 1990 (not seen), all of which, like Allen 1955, suffer from the complex constructs of outdated linguistic systems. No such defects disfigure Shefts 1961 who treated (2) and (3) and was reviewed in Staal 1963a (reprinted in 1988: 232-237).

Before we begin I must mention that Sanskrit does not use an explicit pronoun like English. The verbal form *khādati* does not mean: "eats" but "he eats". That "he" disappears when there is a subject as in *rāmo khādati* which means "*Rāma* eats" and not "*Rāma* he eats" (where the asterisk express ungrammatically) What is important in our context is that Sanskrit distinguishes like English between the

stem and the suffix or ending of a verb. From the stem *khād-* ("eat-") and the ending *-ti* ("-s") it forms:

khād-a-ti "(he) eat-s" (1)

What is *-a-* ? It is something in between which we may call an infix. I put hyphens between the three elements in the Sanskrit expression to distinguish the stem and the ending, which we find in both Sanskrit and English, and highlights the infix we only find in Sanskrit.

There are, however, various classes of verbs in Sanskrit. One of them has a verb with the same meaning but lacks the infix:

ad-ti "(he) eat-s" (2)

which becomes *atti* which is due to what is sometimes called "assimilation". Pāṇini's grammar is a list of rules (*sūtra*). (1) follows from a general rule. (2) seems to illustrate a special case. However, (1) and (2) express similar properties and the underlying rule is the same if we adopt a principle called *lāghava*, literally "lightness". (2) is now expressed as:

ad-0-*ti* "(he) eat-s" (3)

The symbol "0" which indicates absence of the infix is the *lopa* defined as "something that does not appear". Here we meet the grammatical or linguistic zero. It occurs not only in verbs but also in nouns and it should be obvious that there are many of them.

Pāṇini's rules are generally ordered and he could have reversed the order, i.e., start with (2) and derive (1) by inserting the infix. Such problems are discussed by commentators and other grammarians, including Joshi and Kiparsky 1979 and Kiparsky 1991 who have shown that "lightness" is not simply an abbreviation but expresses generalization. It explains the famous saying: "grammarians rejoice over the saving of half a syllable as over the birth of a son". My examples do no more than illustrate the thesis, that the many linguistic zeroes of Sanskrit grammar led mathematicians to muse about one or more mathematical zeroes.

Indic mathematicians did not only study Pāṇini to compose works with all-India appeal. They were a small group of people, not popular or prestigious like mathematicians in the modern world. In India there was only one "science of the sciences" (*śāstrāṇām śāstram*), the boundless (*anantapāram*), the summit of learning: grammar or *vyākaraṇa* which literally means "analysis". Mathematicians were flattered to be associated with such a prestigious tradition. In the modern world, the opposite holds. Grammar is not a popular subject and many scientists do not even know what "linguistics" means. It developed as a serious discipline only after the discovery of Sanskrit by William Jones, Charles Wilkins'

Sanskrit grammar and Franz Bopp's adoption of the methods of the Indian grammarians in his "Systems of Conjugation in Sanskrit compared with those in the Greek, Latin, Persian and Germanic Languages" of 1808 (see Thieme 1982/83, Staal 1989 and 1993, Chapters 4 and 5, and 1995: 109).

The histories of linguistics, logic and the theory of ritual in India and Europe were first compared in Staal 1988: 36-45. It presented graphs of developments that gave a rough idea of the ups and downs. Logic included mathematical logic, but I did not then, and would not now be able to include mathematics. "The Theory of Definition in Indian Logic" (1961, reprinted in 1988:90) referred to the occurrence of a kind of null-class presupposed in "modern" works such as the *Tarka-saṃgraha* of the seventeenth century AD. But in India, linguistics and logic were closely connected almost from the beginning.

Pāṇini distinguished different zeroes from each other by making use of a rich conceptual apparatus. He was aware of the fact that the language of his grammar was modeled in part after the language that was the object of his study: Sanskrit. It could lead to confusion unless the two were clearly distinguished. He therefore made a distinction between "rules" (*sūtra*) and "metarules" (*paribhāṣa*). Implicitly working with what we would now call a metalanguage, Pāṇini made explicit use of meta-linguistic markers which he called *it*. To distinguish the different zeroes from each other, he made use of the fact that *lopa* comes from a verbal root that starts with "l" and to which we shall return. The meta-linguistic markers always have an "l" that marks them as dealing with zeroes such as *luk*, *laṭ*, *liṭ* and *ślu*, each defined for particular classes or special cases. In modern transliterations, they are indicated by capitals (which Sanskrit has no means of distinguishing from small letters). The uses of metalanguage in Sanskrit grammar have been studied by Scharfe 1961 and Staal (1963b, 2003: 353-6); rules and metarules arestudied separately in Staal *forthcoming*.

English grammar does not use such meta-linguistic markers but it could do something similar. It may be illustrated with the help of a rough sketch of English noun pluralization (a formalized grammar of such a topic may look quite different and require a substantial book). We shall begin with a general rule, where *P* is the plural marker:

$$\text{noun} + P > \text{noun} + \text{suffix (e)s}$$

This is a context-sensitive rule in which > stands for "is replaced by"; + stands for concatenation; and parentheses express options that distinguish *dogs* from *witches*. The general rule as stated does not account for *fish* or *sheep* which require a zero-suffix.

My account, so far, applies to written English. It does not explain different pronunciations of the written *s*, which may sound like "*s*" or "*z*". If we try to account for pluralization in both written and spoken English we need a greater

variety of expressions. I shall not belabor the point but Pāṇini's way with meta-linguistic zero-suffixes for special cases may be illustrated again for English by using subscripts as linguistic markers, for example:

$$\begin{aligned} man + 0_1 &> men \\ woman + 0_2 &> women \\ mouse + 0_3 &> mice. \end{aligned}$$

The spelling of English is idiosyncratic but that of Sanskrit, in that respect closer to Italian or Spanish, is rational. It is adopted by all Indic syllabaries in South and Southeast Asia and in the Roman transliteration adopted by Sanskrit scholars worldwide. That transliteration writes the *ou* of English *mouse* as *au* and the *i* of *mice* as *ai*. They are part of an extended system with similar sound correspondences in Sanskrit and Indo-European. Sanskrit derives from nouns such as *śiva* the adjective *Śaiva* which, in English, became "Shivaite". Similarly, the noun *rudra* produces the adjective *raudra* to which no English adjective corresponds.

4. Zeroes in the Śrauta Ritual

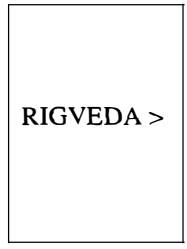
I shall end our discussion with the Vedic Śrauta ritual which belongs to the prehistory of zero as well as that of Sanskrit Grammar. Vedic ritual is, therefore, a parent as well as a grandparent of zero. All forms of Vedic ritual are concerned with recitations, chants and acts. Recitations come from the Rigveda, chants from the *Sāmaveda* and ritual acts are the chief concern of the Yajurveda. Vedic ritual became a science or theory (Staal, *forthcoming*) in one of its later forms which developed between roughly the tenth and seventh centuries BCE and became known as the Śrauta ritual. It was an oral tradition.

Śrauta ritual was the most creative of Vedic rituals. By that time, practice and theory had become highly developed disciplines and were closely connected with each other. That development did not take place among other higher animals which perform rituals but do not have theories because they do not possess language. Human ritualists are able to talk about ritual with each other, introduce modifications and add refinements. They started thinking ritually; but it did not happen often. Śrauta ritualists may have been the first. Japanese theologians influenced their rituals during recent centuries (Sharf 2003). Modern students have their own rituals but remain a motley crowd consisting of theologians, psychologists, representatives of disjointed human and social sciences while ritualization among the higher animals, again including the human, continues to be taken care of by life sciences on the borderline. Many of these specialists are innocent of theory. It explains an important difference between sections 3 and 4.

Sanskrit grammar deals with Saṅskrit and influenced modern linguistics. The *Śrauta* ritual did not influence any modern theory of ritual because there is no such theory. It implies that we cannot adopt Needham's rule as formulated on page 3 above: we cannot do "the only thing we can do" because there is no yardstick. The reader will accordingly meet with unfamiliar concepts, methods and modes of analysis. Some are discussed in Chapters 7, 12 and 13 with their Source Notes in Staal 2008. But unless we are acquainted with one of the few surviving traditions of Śrauta ritual and/or some of the literature in Sanskrit and in modern languages about Vedic ritual, we shall be on our own.

Basic to any ritual performance are space and the four directions. The ritual arena consists of several sheds with thatched roofs that are temporarily constructed for each performance. Some of the most important *Śrauta* ceremonies are performed in a small space at the center that is called the Sadas. The word is derived from the Sanskrit verb *sad-* or "sit" which occurs in the contemporary term *upaniṣad*, "sitting close to" (the teacher).

The Sadas looks as follows with the north on top:



Vedas are recited in the four directions which the reciters themselves must also face. The above sketch makes use of two directions that are indicated in the figure by symbols we have used before but that now have a new meaning: > means "facing east" and < means "facing west". These directions raise a host of technical and theoretical problems, some of them discussed by Caland and Henry (1906-7: 232) and Keith (1914: I, 252 *note* 4). I shall mention two. The first is concerned with the directions only. The second combines directions with the verbal root from which *lopa* derives.

The first case is illustrated by the sketch. It depicts a change of directions that has just taken place. It does not involve the Rigveda which is recited inside the Sadas by an officiant who is already sitting there, facing and reciting east. I shall call him *R*. The Yajurveda is recited by a priest I shall call *Y*, but he has come from outside the Sadas and cannot easily enter. He has made several turns already and will eventually face west and face *R*: the auspicious result that is depicted here. Earlier, the two officiants did not only fail to face each other but *Y* sat with his back to *R* - a situation that is to be avoided at all costs. The entire episode illustrates how the rivalry between different factions may be resolved and overcome.

The stage is now set for the second case. A dialogue unfolds which is initiated by *R* who recites a proposal:

"Let us both recite!" (*śoṃsāvo*)

Y responds from outside the Sadas with a touch of flattery. His verse consists of two halves:

"Let us both recite, divine one!" (*śoṃsāvo daiva*)

"Recite! Let us both rejoice!" (*śaṃsa madeva*)

Suppose *Y* were to omit the second half-verse. I have never witnessed it but it must have happened often because the two halves are very similar. The problem is addressed by the Yajurveda: "if the response after the half-verse were omitted (*lupyeta*) it would be like someone being left behind by others who are running ahead" (*Taittirīya Saṃhitā* 3.2.9.5). Here we have an instance of the verbal root *lup-* from which *lopa* is derived. The general meaning of the verb is "disappear" or "get lost".

Louis Renou, whom I mentioned before, was the first to draw attention to the numerous ritual uses of *lup-* in his 1941-1942 study on the connections between Sanskrit grammar and Vedic ritual (465, *note* 83). It is also Renou, "the most complete Sanskritist" as he was called by V. Raghavan (1956: 20), who first demonstrated the historical precedence of ritual over grammar.

I conclude that ancient India reverberated with zeroes, zero entities and zero events long before the geometry of the *Śulva Sūtras* which are post-Buddhist. What may be called the prehistory of zero was expressed in early Vedic by *kha* which refers to cavities of various sorts and occurs in the *Upaniṣads* in the sense of "space". The *Śrauta Sūtras*, late Vedic but pre-Buddhist, used *lopa* to refer to omissions, disappearances and things that are lost. It is here that the origins of the mathematical concept of zero seem to lie. We do not know where it happened if it happened only once, but the most likely place would be the Kuru region north of modern Delhi though it may have been further east in Magadha, which overlaps with modern Bihar. The time must have been after 1,000 and before 600 BCE when the creative period of the Śrauta ritual was over. It is a long period with smudgy edges but there it is.

Reverting finally to arithmetics there are important questions that I have not so far considered. How was zero conceived as a member of the number series? Takao Hayashi has suggested that it may be related to additions such as 15 + 20 = 35 (in modern symbols) which presuppose 5 + 0 = 5 plausible enough but when did it happen? Or could zero be related to the recursive principle underlying the decimal number names (and hence their construction) which, according to P.P. Divakaran, was perfectly well understood in the early Rigveda? Plausible also, but would it imply that zero was conceived as the beginning of the infinite series of natural numbers, so that one would count 0, 1, 2, 3 etc.?

Let us return once more to our explanation of the origin of the mathematical zero in terms of the assignment of special meanings to Sanskrit terms such as *lopa*, *śunya* or *bindu*. We are fortunate to possess the records of such events. Similar words occur in other human languages but did not undergo a similar development as far as I know. In their Indic evolution, *lopa* was inspired by ritual but is that a necessary part of its prehistory? All we know is that it added the flesh of another empirical discipline to the bones of linguistics, the discipline that underlies our understanding of the development of language. As far as I can tell, thus far, two conditions must be satisfied before a concept of zero may arise: there needs to be a language as well as another formal structure in which that language is used to signify that something has disappeared or is lost.

Acknowledgments

It is with great pleasure that I like to express my gratitude to three colleagues who have assisted me substantially with this paper and of whom the first has already been mentioned: P. P. Divakaran, Sitabhra Sinha and Peter Vandemoortele.

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Frits Staal
Professor Emeritus, University of California at Berkeley.

67 Moo 1, Ban Pong
Hang Dong, Chiang,
Mai Thailand 50230
E-mail: fritsstaal@berkeley.edu