Nanda era. But since the passage quoted above from Mr. Fleet's inscription is beyond suspicion, I must venture to maintain my belief, until further inquiries confirm this view which is forced upon me, or refute it.

Oxford. DR. E. LEUMANN.

AN ADEN EPITAPH.

An epitaph has been discovered in a mosque at Aden, dated A.H. 563 (A.D. 1168). It is supposed to have been brought from one of the disused burial-grounds of Aden, and commemorates "a virtuous free woman the mother of Abdallah the emancipated slave of the glorious Sultân Yehia bin Abi-s-sadâd al Muwaffak al Thagari al Islâmi. Died at Awân on the last day of Ramadhân in the year 563." It is "inscribed by Muḥammad bin Barakàt bin Ali Harami."

Awân is perhaps the old name of Aden itself; at any rate it was almost certainly in the immediate vicinity. The Harami tribe still exists in Hadhramaut.

ASIATIC SOCIETIES.

The *Journal* of the Asiatic Society of Bengal is rather falling into arrears. No. 2 of the volume for 1882 having only been published in September last. It is mostly occupied with a continuation of Babu Sarat Chandradâs's contributions on the Religion, History, &c. of Tibet. These papers are interesting, and it is only to be regretted that the author does not prepare his work with more attention to details of uniformity of spelling, clearness of statement, &c., or that the papers are not more carefully edited. The contributions here presented contain: (1) The rise and progress of Jiñ or Buddhism in China, translated from the *Dub-thah śelkyi Mèlöñ*, prefaced by short accounts of the Mè-tse, Li-ye-tse, Chwân-tse sects, and that of Yu-su, which preceded Buddhism in China and somewhat resembled it; then comes the usual account of the introduction of Buddhism from India; the contributions to its literature, &c. which it received from Tibet; the five Buddhist schools in China, *viz* —1. The Vinaya or Hînayâna. 2. The Mantra or Tântrika; 3. The Vaipulya-Darśana or Mahâyâna; 4. The Gabhira-Darśana or Śûnyatâ; and, 5. The Sârârtha-Darśana schools. (2) The sacred literature and philosophy of ancient China, translated from the same source; the Bon (Pon) religion in China; and the Ho-u-se or Hoi-hoi, apparently a Muhammadan sect, of which the Tibetan author seems to have had but a low opinion:—he says, "they send the spirits of all animals killed by them to The-pan, who takes charge of them. The spirits of those that are killed by others, who are not Hoi-hoi, are damned. A Hoi-hoi will not eat the flesh of an animal that has been slain by outsiders;" and, he adds, "these wicked people certainly turn into pigs after their death, for which reason they do not touch pork, the touch of which brings defilement, and the eating of which destroys their intellect and understanding." (3) The life and legend of Nâgârjuna, the founder of the Mâdhyamika school. According to this account he was the only son of a Brâhman of Vidarbha, whose death the astrologers predicted in a week unless a hundred Bhikshus were fed and religious ceremonies gone through, and even then he would die in his seventh year. Avalôkitêśvara-Khasharpaṇa, however, appeared to him and advised him to go to Nâlendra, where he would escape death. There he was ordained a Bhikshu by the high priest Srî-Saraha-Bhadra, whom he afterwards succeeded. Vajrâsana or Buddha-Gayâ was then the head-quarters of the Śrâvakas—as the decaying Hînayâna sect was then called, and Nâlendra of the Mahâyâna school. He surrounded the great temple of Mahâgandhôla or 'the mansion of fragrance,' with a stone railing, which he furnished with Vajragavâksha or 'precious riches,' and outside of which he erected 108 smaller chapels. He also surrounded the great shrine of Śrî-Dhânyakaṭaka with railings. At this period, "Mañja, king of Otisha (Orissa), with a thousand of his subjects embraced Buddhism." In Mâlvâ, "in the city of Dhârâ, king Bhôjadêva with many hundreds of his subjects embraced Buddhism." He erected "many vihâras in Pratâpêśa, Oṭisha, Bangala, and the country of Ikshuvardhana. In the latter part of his life Nâgârjuna visited Dakshina, where he did many things for the preservation of the Southern congregation." In Drâviḍa he overcame in a disputation two famous Brâhmaṇs—Madhu and Supramadhu—who became converts. He is said to have been a great friend of king De-chye (Śaṁkara), of Southern India, with whom he entered into a compact to live and die. The king's life was thus secured by the saint's; but in this king's old age the mother of the heir-apparent advised her son to ask Nâgârjuna for his head. This he did, and the saint showed him he could only be killed with a blade of *Kuśa* grass. This is followed by (4) Detached notices of different Buddhist schools in Tibet.

The other paper is the first part of one by Mr. Grierson on Manbodh's *Haribans*, containing the text of a Maithilî poem, by a poet named Manbodh or Bholan Jhâ, who died about A.D. 1788. The interest of this is purely philological.

The *Proceedings* of the same Society also is in arrears, the number for July and August appearing only in December. The most important notices in it are:—Lieut.-Col. G. E. Fryer's argument for the date of the Pâlî grammarian, Kachchâyana, being about the 12th century A.D., and Dr. Hoernle's contention that it is really much earlier; and an account of a very ancient fragment of a MS. on Arithmetic found at Bakhshâlî in the Yusufzai district, written in Śâradâ characters and in the Gâthâ dialect, by Dr. Hoernle, which we extract:—

Dr. Hoernle exhibited at the meeting of the Society on 2nd August last a remarkable birch-bark Manuscript, found at Bakhshâlî, in the Yusufzai District, in the Panjâb.

The MS., he said, was found in a ruined enclosure, near Bakhshâlî, a village of the Yusufzai District, in the Panjâb, by a man who was digging for stones. It is written on leaves of birch-bark, which have become so dry by age as to be like tinder, and, unless very carefully handled, they crumble into pieces. Hence, unfortunately, by far the largest portion of the MS. was destroyed when the finder took it up: and even the small portion that now remains is in a very mutilated state. With much care and trouble I have succeeded in separating all the leaves, and have found that 66 of them still remain, of none of which, however, much more than one-half is preserved. For permanent preservation, I mounted each leaf separately between two pieces of 'talc.'

The MS. is written in the so-called Śâradâ characters, which are still used in Kashmir, and which, as they occur on the coins of the Mahârâjas of Kashmir, are of a not inconsiderable age. Some of the forms, which very frequently occur in the MS., especially of vowels, very closely resemble the forms used in the Aśoka and early Gupta inscriptions. I have not observed these particular ancient forms in other MSS. written in the Śâradâ characters, *e.g.*, in the *Mahârṇava* MS. published in the *Cambridge Palæographic Series*. Hence I am inclined to look on them as an evidence of great age in the Bakhshâlî MS.; and as the West Indus Districts were early lost to Hindû civilization through the Muhammadan conquests, during which it was a common practice to bury MSS. to save them from destruction, the Bakhshâlî MS. may be referred to the 8th or 9th century A.D.

I have looked over all the leaves of the MS. that remain, and have carefully read and transcribed about one-third. I have thus seen enough of the fragment to make sure that the whole of it treats of Arithmetic (including apparently Mensuration), though incidentally a few rules of Algebra are noticed. The latter refer to the solution of indeterminate problems (*kuṭṭaka*). The arithmetical problems are of various sorts; *e.g.*, on velocity, alligation, profit and loss, etc. I may give one or two examples thus "A and B run 5 and 9 *yojanas* a day respectively, and A is allowed a start of 7 days or 35 *yojanas*; when will A and B meet?" Or, "A and B earn $2\frac{1}{3}$ and $1\frac{1}{2}$ *dînâras* a day respectively; A makes a present of 10 *dînâras* to B; how soon will their possessions be equal?" An example of an algebraical problem is: "A certain quantity, whether 5 be added to it or 7 be subtracted from it, is a square; what is that quantity?" The solution, given in this case, is 11; for $11 \times 5 = 16$ or 4^2, and $11 - 7 = 4$ or 2^2.

The fragment, however, evidently does not contain the whole of the treatise on Arithmetic; for many subjects, commonly treated in Hindû arithmetical works, do not appear to occur in it; and this is confirmed by the numbers of the rules (or *sûtras*, as they are called). The earliest numbered *sûtra* that I have noticed is the 9th, and from internal evidence I conclude,—though the numbers are lost,—that the 7th and 8th rules are also preserved. The latest number I have met is the 57th.

The method observed in the treatment of the problems is as follows: first a rule is given, introduced by the word *sûtra*; next follow one or more examples, introduced by *tadâ*, and stated both in words and in arithmetical notation; the latter is sometimes indicated by the term *sthâpana*; next follows a solution in words, which is always called *karaṇa* "operation"; and lastly comes the proof, generally expressed in notation, and called *pratyâyana* or *pratyaya*. This method differs considerably from that used in other Hindû arithmetical treatises, *e.g.*, in those of Bhâskara and Brahmagupta. The latter also use different terms; instead of *tadâ*, examples are called by them *uddeśa* or *udâharaṇa*; instead of *sthâpana* they have *nyâsa*; *karaṇa* and *pratyâyana* or *pratyaya* are not used at all. The term *sûtra* they employ occasionally, but in most cases they say *karana sûtra*, which latter term may contain a reference to a *karaṇa*-work such as that in the Bakhshâlî MS. There are, also, some differences in the method of notation as used in this MS. and as commonly established. Division is indicated by placing one quantity under another without a line between them; *e. g.*, $\begin{smallmatrix}5\\8\end{smallmatrix}$ ($= \frac{5}{8}$): multiplication, by placing one quantity beside the other; *e.g.*, $\begin{smallmatrix}5\\8\end{smallmatrix}$ 32 ($= \frac{5}{8} \times 32 = 20$); addition, by writing *yu* (abbreviated for *yuta* "added") before or after the additive quantity and placing the latter either *by the side of*, or *below*, the other quantity; *e.g*, 11 5 *yu* or 11 *yu* 5 ($= 11 + 5 = 16$): subtraction, by writing the negative sign + after the subtractive quantity, and placing the latter beside or below the other quantity; *e.g.*, $\left.\begin{matrix}1\\1\\3+\end{matrix}\right\}$ $(= 1 - \frac{1}{3} = \frac{2}{3})$, or 11. 7+ $(= 11 - 7 = 4)$. This negative sign is the most remarkable difference between the Bakhshâlî MS. and the works of Bhâskara and others. The MS. uses a cross + (exactly resembling our modern *plus* sign), while the sign which is commonly used is a dot, placed above the quantity; *e. g.* 11 $\dot{7}$ $(= 11 - 7 = 4)$. I may add that the cipher is used (as in the *Lîlâvatî*) to indicate an unknown quantity, the value of which is sought; *e. g.*, $\begin{matrix}0&5\\1&1\end{matrix}$ *yu mû* $\begin{matrix}0\\1\end{matrix}$ | $\begin{matrix}0&7\\1&1\end{matrix}$ + *mû* $\begin{matrix}0\\1\end{matrix}$ | (for $\frac{x}{1} + \frac{5}{1} = \frac{y^2}{1}$ and $\frac{x}{1} - \frac{7}{1} = \frac{z^2}{1}$; here $x = 11, y = 4, z = 2$; *mû* abbreviated for *mûlada* "square"). It is, however, also employed in the usual way as the tenth figure of the decimal notation. A proportion is expressed thus :— $\begin{matrix}1\\1\end{matrix}$ | $\begin{matrix}13\\6\end{matrix}$ | $\begin{matrix}30\\1\end{matrix}$ *pha* 65 | (for $1 : \frac{13}{6} = 30 : 65$; *pha* abbreviated for *phalaṁ*.) All these peculiarities of method, terminology and notation, differing as they do from those in common use since the time of Brahmagupta (about 628 A.D.) and Âryabhata (about 500 A.D.), whose mathematical treatises are the earliest known, tend to show that the work contained in the Bakhshâlî MS. is more ancient than any of those I have just mentioned.

There is another remarkable feature in the MS., which points in the same direction, namely, the language in which it is written. This is what is now commonly called the Gâthâ dialect, because it was first noticed in ancient Buddhist works (such as the *Lalita Vistara*) written in verses or gâthâs. The term Gâthâ dialect, however, is no more appropriate now, because that dialect is now known to be also used in ancient Buddhist works, which are partly written in prose, such as the *Mahâvastu*, of which M. Senart has just published an excellent edition. However that may be, it is generally admitted that this species of language is a very ancient one. It is a kind of ungrammatical Sanskrit (judged, that is, by the standard of what is commonly called Sanskrit), interspersed to a large extent with ancient Prâkrit or Pâli forms. There is some dispute as to the exact origin, time and locality of this species of ancient irregular Sanskrit. But in all probability it was current in the early centuries just before and after the commencement of the Christian era, as a literary or cultivated form of the ancient Vernacular Prâkrit of North-Western India, in the countries to the east and west of the Indus, till it came to be superseded by the classical Paṇinian Sanskrit. It is this language which is employed in the Bakhshâlî MS. It would be out of place here to enter into philological details; but I may mention that the language of the MS. is marked by all the peculiarities in orthography, etymology, syntax, etc., of the so-called Gâthâ dialect. The evidence of the language, then, would tend to show that the work contained in the Bakhshâlî MS. must be ascribed, in all probability, to the earliest centuries of the Christian era, and further—since the Gâthâ dialect has hitherto only been met with in Buddhist literature,—to a member of the Buddhist community. If the latter supposition be correct, we should have in this MS. the first Buddhist Arithmetical work which, so far as I am aware, has hitherto become known.

There are, further, some specific points in the work contained in the Bakhshâlî MS. which tend to point to a peculiar connection between it and the mathematical portion of the *Brahma Sphuṭa Siddhânta*, the famous astronomical work of Brahmagupta, which was compiled in 628 A. D. Thus an algebraical rule in the MS. occurs in strikingly similar language in Brahmagupta's algebra; again the foreign terms *dînâra* (Latin *denarius*) and *dramma* (Greek *drachme*) occur in both, etc. The mathematical treatise in the Bakhshâlî MS. is undoubtedly older than that of Brahmagupta; but what the exact connection between the two works may be, I am not as yet in a position to say. These are points which require further investigation, in which I am still engaged, and the results of which I hope to have a future opportunity of communicating to the Society. My present remarks are not intended to be more than a preliminary notice of the MS. In conclusion I will only repeat that the questions of the age of the MS. and of the work contained in it are entirely distinct; and that the date of the work is certainly very much earlier than the MS. copy of which this fragment has been found.[1]

No. 3 for 1882 has been published since, and is occupied by a collection of 64 Hindû Folksongs from the Panjâb, with translations and notes by our able correspondent, Lieut. R. C. Temple. The only other paper is a Note by P. N. Bose, B.Sc., on some earthen pots found in the alluvium at Mahêśvara in Nimâr. These vessels had been already noticed by Capt. Dangerfield (Malcolm's *Central India*, vol. II, p. 325). The author would identify Mahêśvara and the neighbouring Maṇḍalêśvara as the Mahîśamaṇḍala to which Aśôka sent the Thero Mahâdêva as a Buddhist missionary; but the other missionaries were all sent to countries, not towns or small districts, and it seems much more probable that Maisûr is meant by Mahîśamaṇḍala.

[1] *Proc. As. Soc. Beng.* Aug. 1882.

THE BAKHSHALI MANUSCRIPT.

BY DR. A. F. RUDOLF HOERNLE.

THE **Bakhshâlî manuscript** was found, as probably the readers of this Journal (*ante*, Vol. XII. p. 89 f.) will recollect, in May 1881, near a village called Bakhshâlî, lying in the Yûsufzâî Subdivision of the Peshâwar District at the extreme North-Western frontier of India.[1] It was dug out by a peasant in a ruined enclosure, where it lay between stones. After the find it was at once forwarded to the Lieutenant-Governor of the Pañjâb who transmitted it to me for examination and eventual publication.

The manuscript is written **in Śâradâ characters** of a rather ancient type, and **on leaves of birch-bark** which from age have become dry like tinder and extremely fragile. Unfortunately, probably through the careless handling of the finder, it is now in an excessively mutilated condition, both with regard to the size and the number of the leaves. Their present size (see Plate[2]) is about 6 by $3\frac{1}{4}$ inches; their original size, however, must have been about 7 by $8\frac{1}{4}$ inches. This might have been presumed from the well-known fact that the old birch-bark manuscripts were always written on leaves of a squarish size. But I was enabled to determine the point by a curious fact. The mutilated leaf which contains a portion of the twenty-seventh *sûtra* shows at top and bottom the remainders of two large square figures, such as are used in writing arithmetical notations. These, when completed, prove that the leaf in its original state must have measured approximately 7 by $8\frac{1}{4}$ inches. The number of the existing leaves is seventy. This can only be a small portion of the whole manuscript. For neither beginning nor end is preserved; nor are some leaves forthcoming which are specifically referred to in the existing fragments.[3] From all appearances, it must have been a large work, perhaps divided into chapters or sections. The existing leaves include only the middle portion of the work or of a division of it. The earliest *sûtra* that I have found is the ninth; the latest is the fifty-seventh. The lateral margins which usually exhibit the numbering of the leaves are broken off. It is thus impossible even to guess what the original number of the leaves may have been.

The leaves of the manuscript, when received by me, were found to be in great confusion. Considering that of each leaf the top and bottom (nearly two-thirds of the whole leaf) are lost, thus destroying their connection with one another, it may be imagined that it was no easy task to read the fragments and arrange them in order. After much trouble I have read and transcribed the whole, and have even succeeded in arranging in consecutive order a not inconsiderable portion of the leaves containing eighteen *sûtras*. The latter portion I have also translated into English.

The beginning and end of the manuscript being lost, both the name of the work and of its author are unknown. The subject of the work, however, is **arithmetic**. It contains a great variety of problems relating to daily life. The following are examples:—"In a carriage, instead of 10 horses, there are yoked 5; the distance traversed by the former was one hundred, how much will the other horses be able to accomplish?" The following is more complicated:—"A certain person travels 5 *yôjanas* on the first day, and 3 more on each succeeding day; another who travels 7 *yôjanas* on each day, has a start of 5 days; in what time will they meet?" The following is still more complicated:—"Of 3 merchants the first possesses 7 horses, the second 9 ponies, the third 10 camels; each of them gives away 3 animals to be equally distributed among themselves, the result is that the value of their respective properties becomes equal; how much was the value of each merchant's original property, and what was the value of each animal?" The method prescribed in the rules for the solution of these problems is extremely mechanical, and reduces the labour of thinking to a minimum. For example, the last mentioned problem is solved thus:—"Subtract the gift (3) severally from the original quantities (7, 9, 10). Multiply

---

[1] See *Proceedings* of the Asiatic Society of Bengal, for 1882, p. 108.

[2] A transcript and explanation of this plate will be found in note 6, on p. 47, at the end of this article.

[3] Thus at the end of the 10th *sûtra*, instead of the usual explanation, there is the following note: *Etam sûtram dvitîya-patrê vivaritâsti*. The second leaf here referred to is not preserved.

the remainders (4, 6, 7) among themselves (168, 168, 168). Divide each of these products by the corresponding remainder $\left(\frac{168}{4}, \frac{168}{6}, \frac{168}{7}\right)$. The results (42, 28, 24) are the values of the 3 classes of animals. Being multiplied with the numbers of the animals originally possessed by the merchants (42 × 7; 28 × 9, 24 × 10), we obtain the values of their original properties (294, 252, 240). The value of the property of each merchant after the gift is equal (262, 262, 262)." The rules are expressed in very concise language, but are fully explained by means of examples. Generally there are two examples to each rule (or *sûtra*), but sometimes there are many; the twenty-fifth *sûtra* has no less than fifteen examples. The rules and examples are written in verse; the explanations, solutions, and all the rest are in prose. The metre used is the *ślôka*.

The subject-matter is divided in *sûtras*. In each *sûtra* the matter is arranged as follows: First comes the rule, and then the example introduced by the word *udâharaṇa*.* Next, the example is repeated in the form of a notation in figures, which is called *sthâpana*. This is followed by the solution which is called *karaṇa*. Finally comes the proof, called *pratyaya*. This arrangement and terminology differ somewhat from those used in the arithmetic of Brahmagupta and Bhâskara. Instead of simply *sûtra*, the latter use the term *karaṇa-sûtra*. The example they call *uddêśaka* or *udâharaṇa*. For *sthâpana* they say *nyâsa*. As a rule they give no full solution or proof, but the mere answer to the problem. Occasionally a solution is given, but it is not called *karaṇa*.

The **system of notation** used in the Bakhshâlî arithmetic is much the same as that employed in the arithmetical works of **Brahmagupta** and **Bhâskara.**[5] There is, however, a very important exception. The **sign for the negative** quantity is a cross (+). It looks exactly like our modern sign for the positive quantity, but it is placed after the number which it qualifies. Thus $\begin{smallmatrix}12\\1\end{smallmatrix}\ \begin{smallmatrix}7+\\1\end{smallmatrix}$ means 12 − 7 (*i. e.* 5). This is a sign which I have not met with in any other Indian arithmetic, nor, so far as I have been able to ascertain, is it now known in India at all. The sign now used is a dot placed over the number to which it refers. Here, therefore, there appears to be a mark of great antiquity. As to its **origin** I am unable to suggest any satisfactory explanation. I have been informed by Dr. Thibaut of Benares, that Diophantos in his Greek arithmetic uses the letter ψ (short for ἔλλειψις) reversed (thus ⋔) to indicate the negative quantity. There is undoubtedly a slight resemblance between the two signs; but considering that the Hindus did not get their elements of the arithmetical science from the Greeks, a native Indian origin of the negative sign seems more probable. It is not uncommon in Indian arithmetic to indicate a particular factum by the initial syllable of a word of that import subjoined to the terms which compose it. Thus addition may be indicated by *yu* (short for *yuta*), *e. g.*, $\begin{smallmatrix}5\\1\end{smallmatrix}\ \begin{smallmatrix}7\\1\end{smallmatrix}$ *yu* means 5 + 7 (*i. e.* 12). In the case of substraction or the negative quantity, *riṇa* would be the indicatory word and *ri* the indicatory syllable. The difficulty is to explain the connection between the letter *ri* (ऋ) and the symbol +. The latter very closely resembles the letter *k* (क) in its ancient shape (+) as used in the Aśôka alphabet. The only plausible suggestion I can make is, that it is the abbreviation (*ka*) of the word *kanita* 'diminished,' from the root *kanaya*, with which the well-known words *kanîyas*, 'younger' *kanishtha* 'youngest,' *kanyâ* 'maiden,' '*kana* or *kaṇa* 'a small piece,' etc., are connected. It is true the occurrence of the participle *kanita*, as far as I am aware, is not authenticated in the existing Sanskrit literature. But it would be a regular formation, and might have been in use in the old North-Western Prâkṛit of the Buddhists or Jains (see below). Another suggestion is, that the sign represents the syllable *nû* (Prâkṛit for *nyû*), an abbreviation of *nyûna*, 'diminished.' The *akshara* for *nû* (or *nu*) in the Aśôka characters would very closely resemble a cross (+). The difficulty about these and similar suggestions is to account for the retention of an obsolete graphic symbol in the case of the negative sign only. If the sign is really the old symbol for *ka*, its retention

* This word is almost uniformly abbreviated *udâ*. Owing to the graphic symbols for *u* and *ta* being indistinguishable, I at first took the word to be complete and read it *tdâ*. But quite lately I found on a fragment, which had hitherto escaped my notice, the word written in full *udâharaṇa*.

[5] See Colebrooke's *Dissertation on the Algebra of the Hindus*, in his *Essays*, Vol. II. pp. 337 ff.

might perhaps be explained by the fact, that, in its transfer to the Śâradâ alphabet, the letter *ka* has suffered less change of form than many others of the old Aśôka characters. However, for the present, the question must be left an open one.

A whole number, when it occurs in an arithmetical operation, as may be seen from the above given examples, is indicated by placing the number 1 under it. This, however, is a practice which is still occasionally observed in India. It may be worth noting that the number 1 is always designated by the word *rûpa* :[6] thus *sarûpa* or *rûpâdhika* 'adding one,' *rûpôna* 'deducting one.' The only other instance of the use of a symbolic numeral word is the word *rasa* for 'six,' which occurs once in an example in the fifty-third *sûtra*.

The following statement, from the first example of the twenty-fifth *sûtra*, affords a good example of the system of notation employed in the Bakhshâlî arithmetic :—

$$\left|\begin{array}{cccc} \bullet & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ & 3+ & 3+ & 3+ \end{array}\; bh\acute{a}\ 32 \;\right| \; phala\dot{m}\ 108$$

Here the **initial dot** is used very much in the same way as we use the letter *x* to denote the unknown quantity the value of which is sought. The number 1 under the dot is the sign of the whole (in this case, unknown) number. A fraction is denoted by placing one number under the other without any line of separation; thus $\begin{smallmatrix}1\\3\end{smallmatrix}$ is $\frac{1}{3}$, *i. e.* one-third. A mixed number is shown by placing the three numbers under one another: thus $\begin{smallmatrix}1\\1\\3\end{smallmatrix}$ is $1 + \frac{1}{3}$ or $1\frac{1}{3}$, *i. e.* one and one-third. Hence $\begin{smallmatrix}1\\1\\3+\end{smallmatrix}$ means $1 - \frac{1}{3}$ (*i. e.* $\frac{2}{3}$). Multiplication is usually indicated by placing the numbers side by side; thus

$$\begin{array}{cc} 5 & 32 \\ 8 & 1 \end{array}\Big| \; phala\dot{m}\ 20,$$

means $\frac{5}{8} \times 32 = 20$. Similarly $\begin{smallmatrix}1 & 1 & 1\\1 & 1 & 1\\3+ & 3+ & 3+\end{smallmatrix}$ means $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ or $(\frac{2}{3})^3$, *i. e.* $\frac{8}{27}$. *Bhá* is an abbreviation of *bhága*, 'part,' and means that the number preceding it is to be treated as a denominator. Hence $\begin{smallmatrix}1 & 1 & 1\\1 & 1 & 1\\3+ & 3+ & 3+\end{smallmatrix}$ *bhá* means $1 \cdot \frac{8}{27}$ or $\frac{27}{8}$. The whole statement, therefore

$$\left|\begin{array}{cccc} \bullet & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ & 3+ & 3+ & 3+ \end{array}\; bh\acute{a}\ 32 \;\right| \; phala\dot{m}\ 108$$

means $\frac{27}{8} \times 32 = 108$, and may be thus explained,—"a certain number is found by dividing with $\frac{8}{27}$ and multiplying with 32: that number is 108."

The dot is also used for another purpose, namely as one of the ten fundamental figures of **the decimal system of notation,** or the **zero** (0 1 2 3 4 5 6 7 8 9). It is still so used in India for both purposes, to indicate the unknown quantity as well as the naught. With us the dot, or rather its substitute the circle (0), has only retained the latter of its two intents, being simply the zero figure, or the 'mark of position' in the decimal system. The Indian usage, however, seems to show how the zero arose, and that it arose in India. The Indian dot, unlike our modern zero, is not properly a numerical figure at all. It is simply a sign to indicate an empty place or a hiatus. This is clearly shown by its name *śûnya* 'empty.' The empty place in an arithmetical statement might or might not be capable of being filled up, according to circumstances. Occurring in a row of figures arranged decimally or according to the 'value of position,' the empty place could not be filled up, and the dot therefore signified 'naught,' or stood in the place of the zero. Thus the two figures 3 and 7, placed in juxtaposition (37) mean 'thirty-seven,' but with an 'empty space' interposed between them (3 7), they mean 'three hundred and seven.' To prevent misunderstanding the presence of the 'empty space' was indicated by a dot (3 • 7); or by what is now the zero (307). On the other hand, occurring in the statement of a problem, the 'empty place' could be filled up, and here the dot which marked its presence, signified a 'something' which was to be discovered and to be put in the empty place. In the course of time, and out of India, the latter signification of the dot was discarded; and the dot thus became simply the sign for 'naught' or the zero, and assumed the value of a proper figure of the decimal system of notation, being the 'mark of position.' In its double signification, which

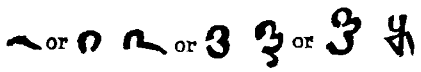
[6] This word was at first read by me *ûpa*. The reading *rûpa* was suggested to me by Professor A. Weber of Berlin and it is, I have now no doubt, the correct one.

still survives in India, we can still discern an indication of that country as its birthplace.

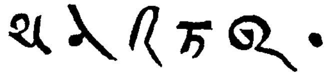
Generally speaking, the terms of an operation are set down side by side: and the particular operation intended is indicated by the initial syllable of a word of that import, subjoined to the terms which compose it. The operation of multiplication alone is not indicated by any special sign. Addition is indicated by *yu* (for *yuta*), subtraction by + (*ka* for *kanita* ?) and division by *bhâ* (for *bhâga*). The whole operation is commonly enclosed between lines (or sometimes double lines), and the result is set down outside, introduced by *pha* (for *phala*). Occasionally the indicatory word is written in full. Vertical lines are usually interposed between the terms of a proportion or a progression. Thus :—

$\left\lvert \begin{matrix}5 & 7\\1 & 1\end{matrix}\ yu \right\rvert$ *pha* 12	means	$5 + 7 = 12$
$\left\lvert \begin{matrix}12 & 7\\1 & 1\end{matrix}\ + \right\rvert$ *pha* 5	„	$12 - 7 = 5$
$\left\lvert \begin{matrix}5 & 32\\8 & 1\end{matrix} \right\rvert$ *pha* 20	„	$\frac{5}{8} \times 32 = 20$
$\left\lvert \dot{1}\ \begin{matrix}1 & 1 & 1\\1 & 1 & 1\\3+ & 3+ & 3+\end{matrix}\ bh\hat{a}\ 32 \right\rvert$ *pha* 108	„	$(1 : \frac{8}{27}) \times 32 = 108$
$\left\lvert \begin{matrix}10\\1\end{matrix} \middle\vert \begin{matrix}30\\1\end{matrix} \middle\vert \begin{matrix}4\\1\end{matrix} \right\rvert$ *pha* $\begin{matrix}12\\1\end{matrix}$	„	$10 : 30 = 4 : 12$

Regarding **the age of the manuscript,** I am unable to offer a very definite opinion. The composition of a Hindu work on arithmetic, such as that contained in the Bakhshâlî MS., seems necessarily to presuppose a country and a period in which Hindu civilisation and Brâhmaṇical learning flourished. Now the country in which Bakhshâlî lies and which formed part of the Hindu kingdom of Kâbul, was early lost to Hindu civilisation through the conquests of the Muḥammadan rulers of Ghaznî, and especially through the celebrated expeditions of Maḥmûd, towards the end of the 10th and the beginning of the 11th centuries A. D. In those troublous times it was a common practice for the learned Hindus to bury their manuscript treasures. Possibly the Bakhshâlî MS. may be one of these. In any case it cannot well be placed much later than the 10th century A. D. It is quite possible that it may be somewhat older. The Śâradâ characters used in it, exhibit in several respects a rather archaic type, and afford some ground for thinking that the manuscript may perhaps go back to the 8th or 9th century. But in the present state of our epigraphical knowledge, arguments of this kind are always somewhat hazardous. The usual form in which the numeral figures occur in the manuscript are the following :—



1 2 3 4



5 6 7 8 9 0

Quite distinct from the question of the age of the manuscript, is that of **the age of the work** contained in it. There is every reason to believe that the Bakhshâlî arithmetic is of a very considerably earlier date than the manuscript in which it has come down to us. I am disposed to believe that the composition of the former must be referred to the earliest centuries of our era, and that it may date from the 3rd or 4th century A.D. The arguments making for this conclusion are briefly the following :—

In the first place, it appears that the earliest mathematical works of the Hindus were written in the *ślôka* measure ;[7]) but from about the end of the 5th century A.D. it became the fashion to use the *âryâ* measure. Âryabhaṭa c. 500 A.D., Varâhamihira c. 550, Brahmagupta c. 630, all wrote in the latter measure. Not only were new works written in it, but also *ślôka*-works were revised and recast in it. Now the Bakhshâlî arithmetic is written in the *ślôka* measure; and this circumstance carries its composition back to a time anterior to that change of literary fashion in the 5th century A. D.

In the second place the Bakhshâlî arithmetic is written in that peculiar language which used to be called **the Gâthâ dialect,** but which is rather the literary form of the ancient North-Western Prâkṛit (or Pâli). It exhibits a strange mixture of what we should now call Sanskṛit and Prâkṛit forms. As shown by the inscriptions (*e.g.* of the Indo-Scythian kings in Mathurâ) of that period, it appears to have been in general use, in North-Western India, for literary purposes till about the end of the

[7] See Professor Kern's Introduction to the *Bṛhat Saṃhitâ* of Varâhamihira.

3rd century A. D., when the proper Sanskṛit, hitherto the language of the Brâhmanic schools, gradually came into general use also for secular compositions. The older literary language may have lingered on some time longer among the Buddhists and Jains, but this would only have been so in the case of religious, not of secular, compositions. Its use, therefore, in the Bakhshâlî arithmetic points to a date not later than the 3rd or 4th century A. D. for the composition of that work.

In the third place, in several examples, the two words *dînâra* and *dramma* occur as denominations of money. These words are the Indian forms of the Latin *denarius* and the Greek *drakhmé*. The former, as current in India, was a gold coin, the latter a silver coin. Golden *denarii* were first coined at Rome in B. C. 207. The Indian gold pieces, corresponding in weight to the Roman gold *denarius*, were those coined by the Indo-Scythian kings, whose line, beginning with Kadphises, about the middle of the 1st century B. C., probably extended to about the end of the 3rd century A.D. Roman gold *denarii* themselves, as shown by the numerous finds, were by no means uncommon in India, in the earliest centuries of our era. The gold *dînâras* most numerously found are those of the Indo-Scythian kings Kanishka and Huvishka, and of the Roman emperors Trajan, Hadrian and Antoninus Pius, all of whom reigned in the 2nd century A.D. The way in which the two terms are used in the Bakhshâlî arithmetic seems to indicate that the gold *dînâra* and the silver *dramma* formed the ordinary currency of the day. This circumstance again points to some time within the three first centuries of the Christian era as the date of its composition.

A fourth point, also indicative of antiquity, which I have already adverted to, is the peculiar use of the cross (+) as the sign of the negative quantity.

There is another point which may be worth mentioning, though I do not know whether it may help in determining the probable date of the work. The year is reckoned in the Bakhshâlî arithmetic as consisting of 360 days. Thus in one place the following calculation is given :—"If in $\frac{800}{727}$ of a year, $2982\frac{486}{727}$ is spent, how much is spent in one day ?" Here it is explained that the lower denomination (*adha-chchheda*) is 360 days, and the result (*phala*) is given as $\frac{1807}{240}$ ($i.e.$ $\frac{2168400}{727} \cdot \frac{727}{800 \cdot 360}$).

In connection with this question of the age of the Bakhshâlî work, I may note a circumstance which appears to point to **a peculiar connection of it with the Brahma-Siddhânta of Brahmagupta.** There is a curious resemblance between the fiftieth *sûtra* of the Bakhshâlî arithmetic, or rather with the algebraical example occurring in that *sûtra*, and the forty-ninth *sûtra* of the chapter on algebra in the *Brahma-Siddhânta*. In that *sûtra*, Brahmagupta first quotes a rule in prose, and then adds another version of it in the *âryâ* measure. Unfortunately the rule is not preserved in the Bakhshâlî MS., but, as in the case of all other rules, it would have been in the form of a *śloka* and in the North-Western Prakrit (or 'Gâthâ dialect'). Brahmagupta in quoting it, would naturally put it in what he considered correct Sanskṛit prose, and would then give his own version of it in his favourite *âryâ* measure.[8] I believe it is generally admitted that Indian arithmetic and algebra, at least, are of entirely native origin. While *Siddhânta*-writers, like Brahmagupta and his predecessor Âryabhaṭa, might have borrowed their astronomical elements from the Greeks or from books founded themselves on Greek science, they took their arithmetic from native Indian sources. Of the Jains it is well known that they possess astronomical books of a very ancient type, showing no traces of western or Greek influence. In India arithmetic and algebra are usually treated as portions of works on astronomy. In any case it is impossible that the Jains should not have possessed their own treatises on arithmetic, when they possessed such on astronomy. The early Buddhists, too, are known to have been proficients in mathematics. The prevalence of Buddhism in North-Western India, in the early centuries of our era, is a well-known fact. That in those early times there were also large Jain communities in those regions, is testified by the remnants of Jain sculpture found near Mathurâ and elsewhere. From the fact of

[8] See note 4 at the end of this article, also note 5.

the general use of the North-Western Prâkṛit (or the 'Gâthâ dialect') for literary purposes among the early Buddhists it may reasonably be concluded that its use prevailed also among the Jains, between whom and the Buddhists there was so much similarity of manners and customs. There is also a diffusedness in the mode of composition of the Bakhshâlî work which reminds one of the similar characteristic observed in Buddhist and Jain literature. All these circumstances put together seem to render it probable that in the Bakhshâlî MS. there has been preserved to us a fragment of an early Buddhist or Jain work on arithmetic (perhaps a portion of a larger work on astronomy), which may have been one of the sources from which the later Indian astronomers took their arithmetical information. These earlier sources, as we know, were written in the *ślôka* measure, and when they belonged to the Buddhist or Jain literature, must have been composed in the ancient North-Western Prâkṛit. Both these points are characteristics of the Bakhshâlî work. I may add that one of the reasons why the earlier works were, as we are told by tradition, revised and re-written in the *âryâ* measure by later writers such as Brahmagupta, may have been that in their time the literary form ('Gâthâ dialect') of the North-Western Prâkṛit had come to be looked upon as a barbarous and ungrammatical jargon as compared with their own classical Sanskṛit. In any case the Buddhist or Jain character of the Bakhshâlî arithmetic would be a further mark of its high antiquity.

Throughout the Bakhshâlî arithmetic the decimal system of notation is employed. This system rests on the principle of the 'value of position' of the numbers. It is certain that this principle was known in India as early as A. D. 500. There is no good reason why it should not have been discovered there considerably earlier. In fact, if the antiquity of the Bakhshâlî arithmetic be admitted on other grounds, it affords evidence of an earlier date of the discovery of that principle. As regards the zero, in its modern sense of a 'mark of position' and one of the ten fundamental figures of the decimal system (0 1 2 3 4 5 6 7 8 9), its discovery, or rather its elaboration, is undoubtedly much later than the discovery of the 'value of position.' It is quite certain, however that the application of the latter principle to numbers, in ordinary writing, would have been nearly impossible without the employment of some kind of 'mark of position,' or some mark to indicate the 'empty place' (*sûnya*). Thus the figure 7 may mean either 'seven' or 'seventy' or 'seven hundred,' according as it be or be not supposed to be preceded by one (7 • or 70) or two (7 • • or 700) 'empty places.' Unless the presence of these 'empty places' or the 'position' of the figure 7 be indicated, it would be impossible to read its 'value' correctly. Now what the Indians did, and indeed still do, was simply to use for this purpose the sign which they were in the habit of using for the purpose of indicating *any* empty place or omission whatsoever in a written composition: that is the dot. It seems obvious from the exigencies of writing that the use of the well-known dot as the mark of an empty place must have suggested itself to the Indians as soon as they began to employ their discovery of the principle of the 'value of position' in ordinary writing. In India the use of the dot as a substitute for the zero must have long preceded the discovery of the proper zero, and must have been contemporaneous with the discovery of the principle of the 'value of position.' There is nothing in the Bakhshâlî arithmetic to show that the dot is used as a proper zero, and that it is anything more than the ordinary 'mark of an empty place.' The employment, therefore, of the decimal system of notation such as it is, in the Bakhshâlî arithmetic, is quite consistent with the suggested antiquity of it.

I have already stated that the Bakhshâlî arithmetic is written in the so-called Gâthâ dialect or in that literary form of the North-Western Prâkṛit which preceded the employment, in secular composition, of the classical Sanskṛit. Its literary form consisted in what may be called (from the Sanskṛit point of view), an imperfect sanskritisation of the vernacular Prâkṛit. Hence it exhibits at every turn the peculiar characteristics of the underlying vernacular. The following are some specimens of orthographical peculiarities :—

Insertion of euphonic consonants: of *m*, in *êka-m-êkatraṁ*, *bhâitakê-m-êkapaṇḍitah*; of *r*, in *tri-r-asti*, *labhatê-r-ashtou*.

Insertion of *s*, in *vibhaktaṁ-s-uttarê*, *kshiyatê*.

s-traya. This is a peculiarity not known to me elsewhere, either in Prâkṛit or in Pâli.

Doubling of consonants: in compounds, *prathama-d-dhântê, êka-s-saṁkhyá*; in sentences *yadi-sh-shaḍbhi, êtê-s-samadhaná.*

Peculiar spellings: *triṅśá* or *triṅśa* for *triṁśat*. The spelling with the guttural nasal before *ś* occurs only in this word; not otherwise, e. g. *chatválimśa* 40. Again *ṛi* for *ri* in *tṛiliné, kṛiyatê, vimiśritaṁ, kriṇâti*; and *ri* for *ṛi* in *rinaṁ, drishṭaḥ.* Again *katthyatáṁ* for *kathyatáṁ.* Again the *jihvámúlîya* and the *upadhmáníya* are always used before gutturals and palatals respectively.

Irregular sandhi: *kô sô rá°* for *kaḥ sa rá,° dvayô kêchi* for *dvayaḥ k°. dvayô cha* for *dvayaś cha. dvibhi kri°* for *dvibhiḥ kri°, ádyô vi°* for *ádyôr vi°, vivaritásti* for *vivaritam asti.*

Confusion of the sibilants: *ś* for *sh*, in *śashṭi* 60, *máśakô*; *sh* for *ś*, in *daśáṁsha, vishôdhayêt, shêshaṁ*; *ś* for *s*, in *sáśyaṁ, sáśyatáṁ*; *s* for *sh*, in *ésa* 'this.'

Confusion of *ṇ* and *n*: *utpaṇṇa* for *utpanna*; *kshayêna* for *kshayeṇa* (s. 27); *viṇyastaṁ* for *vinyastaṁ.*

Elision of a final consonant: *bhájayê, kêchi*, for *bhájayêt, kêchit.*

Interpolation of *r*: *hrîṇaṁ* for *hînaṁ.*

The following are specimens of etymological and syntactical peculiarities :—

Absence of inflection: nom. sing. masc., *êsa sá rási* for *rásiḥ* (s. 50), *gaváṁ visésha kartavyaṁ* for *visêshaḥ* (s. 51); nom. plur., *sêvya santi* for *sêvyáḥ* (s. 53); acc. plur., *dinára dattaván* for *dinárán* (s. 53).

Peculiar inflection: gen. sing., *gatisya* for *gatêḥ* (s. 15); parasm. for âtm., *vikriṇáti* for *vikrîṇîtê* 'he sells' (s. 54), âtm. for parasm., *arjayatê* for *arjayati* 'he earns' (s. 53).

Change of gender: masc. for neut., *múlá* for *múláni* (s. 55); neut. for masc., *vargaṁ* for *vargaḥ* (s. 50); neut. for fem., *yutiṁ cha kartavyá* for *yutiś* (s. 50).

Exchange of numbers: plur. for sing., *(bharêt) lábháḥ* for *lábhaḥ* (s. 54).

Exchange of cases: acc for nom., *dvitiyaṁ arjayatê bhadhaḥ* for *dvitiyaḥ* (s. 53), acc. for instr., *kshayaṁ saṁguṇya* for *kshayêṇa* (s. 27); acc. for loc., *kiṁ kálaṁ* for *kasmin kále* (s. 52): instr. for loc., *anêna kálêna* for *asmin kálê* (s. 53); instr. for nom., *prathamêna dattaván* for *prathamô* (s. 53), or *êkêna yuti* for *êkô* (s. 15); loc. for instr., *prathamê dattá* for *prathamêna* (s. 53), or *mánavê gṛihîtaṁ* for *mánavêna* (s. 57): gen. for dat., *dvitiyasya dattá* for *dvitîyáya* (s. 53).

Abnormal concord: incongruent cases, *ayaṁ prashṭê* for *asmin* (s. 52): incongruent numbers, *êsa lábháḥ* for *lábhaḥ* (s. 54), *rájaputrô kêchi* for *rájaputráḥ* (s. 53); incongruent genders, *sá kálaṁ* for *tat kálaṁ* (s. 52). *visêsha kartavyaṁ* for *kartavyaḥ* (s. 51). *sá rásiḥ* for *sa* (s. 50), *káryaṁ sthitaḥ* for *sthitaṁ* (s. 14).

Peculiar forms: *nivarita* for *nivṛita, rája* for *arjana, divaddha* 'one and one-half,' *chatválimśa* 40, *paṁchâśama* 50th, *chaupaṁchâśama* 54th, *chaturâśîti* 84, *tri-r-âśîti* 83, *piṇyasê* (v. l. *viṇyasê*) for *apinyasêt, bhájayéta* 'let it be divided' for *bhájyêta* (s. 27).

Peculiar meanings: *yadṛichchhá*, or *kámikaṁ* for the 'number one,' when representing the unknown quantity of which the value is sought.

The following extracts may serve as **specimens of the text :—**

TEXT.

18th Sûtra.

Âdyôr viśêshadviguṇaṁ chayaśuddhivibhâjitaṁ |
Rûpâdhikaṁ *tathá kálaṁ* gatisâsyaṁ tadâ bhavêt ||

Udâ ||

Dvayâditrichayaś chaiva dvi*chayatryá*dikôttaraḥ |
Dvayô cha bhavatê paṁthà kêna kâlêna sâsyatâṁ ||

sthâpanaṁ kriyatê | *êshâṁ* â $\frac{2}{1}$ | u $\frac{3}{1}$ | pa 1 |

dvi â $\frac{3}{1}$ | u $\frac{2}{1}$ | pa $\frac{\bullet}{1}$ | karaṇaṁ || âdyôr viśêsha

. . . .

. . . . tâ dvi 2 . . .

Udâ ||

. . . . |

. . . . ||

â $\frac{5}{1}$	u $\frac{6}{1}$	pa $\frac{\bullet}{1}$	dha $\frac{\bullet}{1}$
â $\frac{10}{1}$	u $\frac{3}{1}$	pa $\frac{\bullet}{1}$	dha $\frac{\bullet}{1}$

karaṇaṁ | âdyôr viśesham âdi 5 | 10 *viśêsha* 5 | chayaśuddhi chayaṁ 6 | 3 śuddhi 3 âdiś sha 5 dviguṇaṁ 10 uttaraviśêsha 3 vibhaktaṁ $\frac{10}{3}$ sarûpaṁ $\frac{13}{3}$ êsha pa lam anêna kâlêna samadhanà bhavanti ||

pratyayaṁ || rûpôṇakaraṇ'na phalaṁ | 65 / dvi 65 |

Ashṭhâdaśamasûtraṁ 18 || + ||

27th Sûtra.

Idânîṁ suvarṇakshayaṁ vakshyâmi yasyêdaṁ sûtraṁ |

Kshayaṁ saṁguṇya kanakâs tadyutir-b-bhâjayêt tataḥ |
Saṁyutair êva kanakair êkaikasya kshayô hi saḥ ||

Udâ ||

*Êkadvitrichatus*saṁkhyasuvarṇâ mâshakai riṇai |
Êkadvitrichatussaṁkh*yai* rahita[9] samabhâgatâṁ ||

sthâpanaṁ kriyatê | êshâṁ

1−	2+	3+	4+
1	2	3	4

karaṇaṁ || kshayaṁ saṁguṇya kanakâdibhi kshayêna saṁguṇya jâtaṁ | 1 | 4 | 9 | 16 | *tadyuti* | êsha yuti 30 kanakâ yuti 10 anêna bhaktvâ labdhaṁ

10 / 1	30 / 1	1 / 1	*pha mâśê*	3 / 1
10 / 1	30 / 1	2 / 1	*pha mâśê*	6 / 1
10 / 1	30 / 1	3 / 1	*pha mâśê*	9 / 1
10 / 1	30 / 1	4 / 1	pha mâś	12 / 1

Udâ ||

Êkadvitri*chatussaṁkhya*suvarṇa prôjjhitâ ime |
Mâśakâ dvitritâṁ chaiva chatuḥpaṁchakarâṁśakaṁ[10] kiṁ kshayaṁ ||

1	2	3	4
1	1	1	1
2	3	4	5

karaṇaṁ | kshayaṁ saṁguṇya kanakâ êsha sthâpyatê

1	2	3	4
2	3	4	5

-s-tadyutir-b-bhâjayêta[11] tataḥ harasâsyê kṛitê yutaṁ 163 / 60 | saṁyutaiḥ kanakair bhaktvâ tadâ kanakâ 10 anêna bhaktaṁ jâtaṁ 163 / 600 êsa êkaikasuvarṇasya kshayaṁ || pra*tyayaṁ trairâśik*na kartavya ||

10 / 1	163 / 60	1 / 1	*pha*	163 / 600
10 / 1	163 / 60	2 / 1	*pha*	163 / 300
10 / 1	163 / 60	3 / 1	*pha*	163 / 200
10 / 1	163 / 60	4 / 1	pha	163 / 150

Udâ ||

. . . . |
. . . . śruṇuśva mê ||
Kramêṇa dvaya mâshâdi uttarê êkahînatâṁ |
Suvarṇaṁ mê *tu* sammiśrya katthyatâṁ gaṇakôttama ||

sthâpanaṁ

4+	5+	6−	7−	8+	9+	1+	2+	3+
5	6	7	8	9	10	2	3	4

kshayaṁ saṁguṇya jâtaṁ 20 | 30 | 42 | 56 | 72 | 90 | 2 | 6 |[12]) êshâṁ yuti 330 kanakânâṁ yuti 45 | anêna bhaktvâ labdhaṁ 330 / 45 | paṁchadaśabhâgê-ś-chhêda kriyatê | phalaṁ | 7 śê 1/3 êsa êkaikamâśakakshayaṁ | pratyaya trairâśikêna | 45 / 1 | 330 / 1 | 1 / 1 | phalaṁ 22 / 3 êvaṁ sarvêshâṁ pratyayô *kartavya* ||

Saptaviṁśatimasûtraṁ 27 || + ||

50th Sûtra.

Yutahînaṁ cha-m-êkatvaṁ . . . . |
. . . . *hînê yutîṁ cha kartavyâ*[13] ||

Udâ ||

Kô râśi paṁchayutâ mû*ladaḥ* sâ râśis saptahîna mûlada |
Kô sô râśir iti prashṭaḥ[14] ||

• 5/1 yu mû • 1 sâ • 1 7+/1 mû • 1 karaṇaṁ |

yutahînaṁ cha-m-êkatvaṁ 12 | taddalaṁ | 6 | dvihînaṁ | 4 | dalaṁ | 2 | vargaṁ | 4 | hînayutiṁ cha kartavyâ | *hînaṁ* 7−| anêna yuti | 11 | êsa sâ râśi || asya pratyânayanaṁ *kriyatê*

| 11 / 1 yu 5 / 1 mû 4 / 1 | 11 / 1 7+ / 1 mû 2 / 1 |

Paṁchâśamasûtraṁ || +

[9] The two first letters (*rahi*) are uncertain, owing to a defect in the texture of the leaf.

[10] Read *chatuḥpaṁchâṁśakân kiṁ kshayaṁ*, metri causâ.

[11] Read *bhâjayêta*.

[12] Here | 12 | is omitted in the MS., by mistake.

[13] These fragments of the *sûtra* have been restored from what appear to be quotations in the solution.

[14] There seems to be some confusion about this example. The first line as it stands does not scan; moreover instead of *kô râśi* it should be *yô râśi*. The second half-line does scan; but the words *iti prashṭaḥ* seem out of place as a portion of the verse. Now if we omit *iti prashṭaḥ* from the verse, the remainder, with a few slight alterations, reads as a correct verse of one line and a half, though in utter disregard of all cæsura. Thus:—

Yô râśi paṁchayuta mûladaḥ sâ râśi saptahîne-
na mûlada kô sô râśir [illegible] iti prashṭaḥ

Perhaps that disregard accounts for the confusion made by the scribe of the MS.

51st Sûtra.

Gavâṁ viśêsha kartavyaṁ dhanaṁ chaiva puna . . ।
. . . . ॥

53rd Sûtra.

Ahadravyaharâśau ta[15]) tadviśêshaṁ vibh*ája-yét* ।
Yallabdhaṁ dviguṇaṁ kâlaṁ dattâ samadhanâ prati ॥

Udâ ॥

Tridinê ârjayê paṁcha bhṛitakô-m-êkapaṇḍitaḥ ।
Dvitîyaṁ paṁchad*i*vasê rasam ârjayatê budhaḥ ॥
Prathamêna dvitîyasya sapta dattâni . . taḥ ।
Datvâ samadhanâ jâtâ kena kâl·na katthyatâṁ ॥

$\frac{5}{3}$ *rû* ‖ $\frac{6}{5}$ *rû* ‖ *karaṇaṁ* । *ahadravyaviśêshaṁ* harâṁśau tat tadviś·shaṁ

anêna kâlêna samadhanâ bhavanti ॥ pratyaya trairâśikê kriyatê | $\frac{3}{1}$ | $\frac{5}{1}$ | $\frac{30}{1}$ | pha 50 | prathamê dvitîyasya- | $\frac{5}{1}$ | $\frac{6}{1}$ | $\frac{30}{1}$ | pha 36 | s-sapta dattâ । 7 śêshaṁ 43 ॥ 43 । 43 êtê samadhanâ jâtâ ॥

Udâ ॥

*Rá*japutrô dvayô kêchi nṛipati-s-sêvya santi vaiḥ ।
M-êkâsyâhnê dvaya-sh-shaḍbhâgâ[16]) dvitîyasya divarddhakaṁ ॥
Prathamêna dvitîyasya daśa dînâra dattavân ।
Kêna kâlêna samatâṁ gaṇayitvà vadâśu mê ॥

‖ $\frac{13}{6}$ | $\frac{3}{2}$ ‖ dattaṁ $\frac{10}{1}$ ‖ karaṇaṁ ॥ ahadravyaviś·shaṁ cha । tatra

pratyayaṁ trairâśikêna

| $\frac{1}{1}$ | $\frac{13}{6}$ | $\frac{30}{1}$ | pha 65 | prathamêna dvitîyasya 10 dattâ jâtâ । 55 । 55 ॥ samadhanâ jâtâ ॥ Sûtraṁ tripaṁchâśamaḥ sûtraṁ 53 ॥ + ॥
| $\frac{1}{1}$ | $\frac{3}{2}$ | $\frac{30}{1}$ | pha 45 |

Sûtra.

Udá ॥

. . . . ।

. dviguṇaṁ dvitîyasya prathamâ . . . . . ॥
Prathamâ chaturguṇaṁ chaiva chaturthê chaiva dattavân ।
Cha . . . . śatam êkaṁ dvayânugaṁ ॥
Vadasva prathamê dattaṁ kiṁ pramâṇaṁ . . sya cha ॥

‖ $\frac{\bullet}{1}$ | $\frac{2}{1}$ $\frac{3}{1}$ ‖ $\frac{4}{1}$ ‖ drishya $\frac{200}{1}$ | śûnyam êkayutaṁ kṛitvà 1 | 2 | 3 | 4 | *pra*ksh·payuktyâ phalam ‖ 20 | 40 | 60 | 80 | êvaṁ 200 ‖ êsha *pratyaya* | â $\frac{20}{1}$ | u $\frac{20}{1}$ | pa $\frac{4}{1}$ | rûpôṇakaraṇêna phalaṁ 200 ॥

Sûtra.

Yadṛichchhâ pinyasê śûṇyê tadâ vargaṁ *tu kárayêt* ।
. . . . ॥

Udá ॥

. . . . ।
Tadâ cha triguṇa*ṁ* datta*ṁ* . . . . ॥
. . . . prathamasya tu kiṁ bhavêt ॥

‖ $\frac{\bullet}{1}$ | tadâ $\frac{2}{1}$ ‖ tadâ $\frac{3}{1}$ ‖ *tadá* $\frac{4}{1}$ | *dattaṁ* $\frac{132}{1}$ ‖ *karaṇaṁ* । yadṛichchhà vinyasê śûnyê । tatrêchchhâ ‖ 1 ‖ tadâ vargaṁ tu kârayêt | $\frac{1}{1}$ | $\frac{2}{1}$ | $\frac{2}{1}$ $\frac{3}{1}$ | $\frac{6}{1}$ $\frac{4}{1}$ | *pra*kshipê guṇitaṁ ‖ 1 | 2 | 6 | 24 । *pra*kshiptaṁ 33 ‖ drishyaṁ vibhajêt | $\frac{132}{33}$ | vartyaṁ jâtaṁ | $\frac{4}{1}$ | êsha pratha*mê*ṇa dattaṁ ॥ atô nyâsaḥ ‖ 4 | 8 | 24 | 96 | dattam 132 êsha vargakramagaṇitaṁ ॥ atha yutivargaṁ dva-*triṁśádhika*śataṁ ॥

Kâmikaṁ śûnyavinyastaṁ tadâ chaiva kramê guṇaṁ ।
. . . . ॥

Udá ॥

. . . . ।
. . . . . . kṛitvâ chaturtha . . . . . ।
. . . . prathamasya kiṁ bhavêt ॥

sthápanaṁ | $\frac{\bullet}{1}$ $\frac{2}{1}$ | $\frac{3}{1}$ $\frac{3}{1}$ | $\frac{12}{1}$ $\frac{4}{1}$ | dri 300 | kâmikaṁ śûnyapinyastaṁ *ká*mikaṁ 1 ॥ êsha nyastaṁ pra*thama*râśau । tadâ chaiva kramêṇa guṇitaṁ | 1 | 2 | 9 | 48 | êshâṁ yu*ti* prakshêpaṁ | $\frac{60}{1}$ | anêna drishyaṁ bhâjitaṁ | $\frac{1}{60}$ | $\frac{300}{1}$ | jâtâ | 5 | *êsha pra*thamasya dhanaṁ ॥ anêna

[15] Read °*harâṁśau tat*.

[16] Read *êkasyâhnê drishaḍbhágá*. The error appears to have been noticed by the scribe of the manuscript.

kshêpaṁ guṇayê | 5 | 10 | 45 | 240 | *êvaṁ* 300 *ê*sha yutivargagaṇitaṁ ||

Udâ ||

Prathamasya na *jânâmi* kathaṁ dattaṁ cha vai dhanaṁ |
Sa cha dvyardhayutaṁ dattaṁ . . . . ||

Udâ ||

. . . . |
. . . . dattaṁ chaiva chaturguṇaṁ ||
. . . . śataṁ chatuśchatvâliṁśâdhikaṁ |
Kiṁ prathamasya dhana . . . . ||

$\left|\begin{matrix} & 1\\ \bullet & 1\\ 1 & 2\end{matrix}\right|\begin{matrix} & 2\\ 2 & 1\\ 1 & 2\end{matrix}\left|\begin{matrix} & 3\\ 3 & 1\\ 1 & 2\end{matrix}\right|\begin{matrix} & 4\\ 4 & 1\\ 1 & 2\end{matrix}\left\|\ \text{dṛi}\ \begin{matrix}144\\ 1\\ 2\end{matrix}\right\|$ . . . . śûnyêsu $\left|\begin{matrix} & 1\\ 1 & 1\\ 1 & 2\end{matrix}\right|$ yutaṁ chaiva guṇaṁ tataḥ | yutaṁ chaiva guṇaṁ kṛitvâ kârayê gaṇakraman tu $\frac{5}{2}$ | guṇaṁ | uparê uparaṁ adhê adhaṁ guṇayê | $\frac{10}{2}$ | sârdhadvayayutaṁ | $\frac{15}{2}$ | tritîyarâśyâ guṇanaṁ | sârdhais saptabhi trîṇi | $\frac{45}{2}$ | sârdhatrayayutaṁ | $\frac{52}{2}$ | chaturtharâśi guṇayê-shshaḍviṁśatibhi | jâtâ | $\frac{208}{2}$ | sârdhachatvâriyutaṁ | $\frac{217}{2}$ | *prakshêpayu*ti | $\frac{289}{2}$ | *êvaṁ* dṛiśyaṁ | sarvaṁ tadêva jâtaṁ ||

Udâ ||

. . . . |
. . . . triguṇaṁ trisârdhayutaṁ ||
Chaturguṇaṁ chaturthêna navârdhayutaṁ dattaṁ[17] |
. . . . *dviśatâ* dvâviṁśâdhikâ ||
Kim atra prathamasya dattâsît ?

$\left\|\begin{matrix}\bullet & 3\\ 1 & 2\end{matrix}\right|\begin{matrix}2 & 5\\ 1 & 2\end{matrix}\left|\begin{matrix}3 & 7\\ 1 & 2\end{matrix}\right|\begin{matrix}4 & 9\\ 1 & 2\end{matrix}\Big\|$ êkatraṁ dattaṁ 222 | śûṇyâ sthânê rûpaṁ datvâ || 1 || yutaguṇita yutakramêṇa jâtaṁ || sthâpa*naṁ* | $\frac{5}{2}$ | $\frac{15}{2}$ | $\frac{67}{2}$ | $\frac{357}{2}$ | dṛishya 222 | prakshêpêna jâtaṁ 222 || *da*ttaḥ dṛiśyâh 222 || jâ*taṁ* ||

Udâ ||

Prathamaṁ[18] na jânâmi divardhayutaṁ . . . |

. . ṇaṁ paṁchâśayutaṁ prathama . . . . ||

karaṇaṁ || śûṇyê rûpaṁ datvâ : yutaṁ jâtaṁ $\frac{5}{2}$ | prathamâ *dviguṇaṁ paṁchârdharahitaṁ* | *śêshaṁ* | $\frac{5}{2}$ | prathamâ tṛitîyaṁ triguṇa*ṁ saptârdharahitaṁ* | *śêshaṁ* | $\frac{8}{2}$ | *prath*amâ chaturthaṁ chaturguṇaṁ navârdharahitaṁ | śêshaṁ | $\frac{11}{2}$ | *êsha nyâsaḥ* | $\frac{5}{2}$ | $\frac{5}{2}$ | $\frac{8}{2}$ | $\frac{11}{2}$ | dṛi $\frac{29}{2}$ | prakshêpayuktiḥ | $\frac{29}{2}$ | vibhaktaṁ | $\frac{2}{29}$ | $\frac{29}{2}$ | jâtaṁ | 1 | *anêna* guṇitaṁ tadêva | êvaṁ riṇarâśî bhavanti | triprakâraṁ samâptaṁ || Sûnyasthânê rûpaṁ datvâ | tadanuyuktaṁ | guṇita . .

TRANSLATION.

18th Sûtra.

Twice the difference of the two initial terms, divided by the difference of the (two) increments, and further augmented by one, shall be the time that determines the progression.

First Example.

A person has an initial (speed) of two and an increment of three, another has an increment of two and an initial (speed) of three. Let it now be determined in what time the two persons will meet in their journey.

The statement is as follows :

No. 1, init. term 2, increment 3, period x
No. 2, ,, ,, 3, ,, 2, ,, x

Solution :—" the difference of the two initial terms" (2 and 3 is 1 ; the difference of the two increments 3 and 2 is 1; twice the difference of the initial terms 1 is 2, and this, divided by the difference of the increments 1, is $\frac{2}{1}$, and augmented by 1, is $\frac{3}{1}$; this is the period. In this time [3] they meet in their journey which is 15).

Second Example.

(The problem in words is wanting; it would be something to this effect :—*A* earns 5 on the first and 6 more on every following day; *B* earns 10 on the first and 3 more on every following day; when will both have earned an equal amount ?)

[17] This line is short by one syllable, and otherwise not regular in scanning. The final question appears to be in prose.

[18] Read *prathamasya* metri causâ, as in one of the preceding examples.

Statement :—

No. **1**, init. term 5, increment 6, period x, possession x.

No. **2**, init. term 10, increment 3, period x, possession x.

Solution :—"Twice the difference of the two initial terms," etc.; the initial terms are 5 and 10, their difference is 5. "By the difference of the (two) increments;" the increments are 6 and 3; their difference is 3. The difference of the initial terms 5, being doubled, is 10, and divided by the difference of the increments 3, is $\frac{10}{3}$, and augmented by one, is $\frac{13}{3}$. This (*i. e.* $\frac{13}{3}$ or $4\frac{1}{3}$) is the period: in that time the two persons become possessed of the same amount of wealth.

Proof :—by the *rûpôṇa* method the sum of either progression is found to be 65 (*i. e.* each of the two persons earns 65 in $4\frac{1}{3}$ days).

27th Sûtra.

Now I shall discuss the wastage (in the working) of gold, the rule about which is as follows :—

Having multiplied severally the parts of gold with the wastage, let the total wastage be divided by the sum of the parts of gold. The result is the wastage of each part (of the whole mass) of gold.

First Example.

Suvarṇas numbering respectively one, two, three, four, are subject to a wastage of mâshakas numbering respectively one, two, three, four. Irrespective of such wastage they suffer an equal distribution of wastage. (What is the latter ?)

The statement is as follows :—

Wastage — 1, — 2, — 3, — 4 mâshaka.
Gold 1, 2, 3, 4 suvarṇa.

Solution :—"Having multiplied severally the parts of gold with the wastage," etc.: by multiplying with the wastage, the products 1, 4, 9, 16 are obtained; "let the total wastage," its sum is 30; the sum of the parts of gold is 10; dividing with it, we obtain 3. (This is the wastage of each part, or the average wastage, of the whole mass of gold.)

(Proof by the rule of three is the following) :—as the sum of gold 10 is to the total wastage of 30 mâshakas, so the sum of gold 4 is to the wastage of 12 mâshakas, etc.

Second Example.

There are suvarṇas numbering one, two three, four. There are thrown out the following mâshakas; one-half, one-third, one-fourth, one-fifth. What is the (average) wastage (in the whole mass of gold) ?

Statement :—

quantities of gold, 1, 2, 3, 4 suvarṇa.
wastage $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ mâshaka.

Solution :—"Having multiplied severally the parts of gold with the wastage," the products may thus be stated,—$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. "Let the total wastage be divided;" the division being directed to be made, the total wastage is $\frac{163}{60}$; dividing "by the sum of the parts of gold;" here the sum of the parts of gold is 10: being divided by this, the result is $\frac{163}{600}$. This is the wastage of each part of the whole mass of gold.

Proof may be made by the rule of three :—as the sum of the parts of gold 10 is to the total wastage of $\frac{163}{60}$ mâshaka, so the sum of gold 4 is to the wastage of $\frac{163}{150}$ mâshaka, etc.

Third Example.

(The problem in words is only partially preserved, but from its statement in figures and the subsequent explanation, its purport may be thus restored) :—

Of gold mâshakas numbering respectively five, six, seven, eight, nine, ten, quantities numbering respectively four, five, six, seven, eight, nine, are wasted. Of another metal numbering in order two mâshakas, etc. (*i. e.*, two, three, four) also quantities numbering in order one, etc. (*i.e.*, one, two, three), are wasted. Mixing the gold with the alloy, O best of arithmeticians! tell me (what is the average wastage of the whole mass of mixed gold) ?

Statement :—

wastage : — 4,—5,—6,—7,—8,—9;—1,—2,—3.
gold : 5, 6, 7, 8, 9, 10; 2, 3. 4.

(Solution) :—"Having multiplied severally the parts of gold with the wastage," the products are 20, 30, 42, 56, 72, 90. 2, 6, 12; their sum is 330; the sum of the parts of gold is 45: dividing by this we obtain $\frac{330}{45}$; this is reduced by 15 (*i. e.* $\frac{22}{3}$); the result is 7 leaving $\frac{1}{3}$ (*i. e.* $7\frac{1}{3}$); that is the wastage of each mâshaka (of mixed gold).

Proof :—by the rule of three :—as the total gold 45 is to the total wastage 330, so 1 mâshaka of gold is to $\frac{22}{3}$ parts of wastage. In the same way the proof of all (the other) items is to be made (*i. e.* $45 : 330 = 5 : \frac{110}{3}$; $45 : 330 = 6 : 44$; $45 : 330 = 7 : \frac{154}{3}$; $45 : 330 = 8 ; \frac{176}{3}$; $45 : 330 = 9 : 66$; $45 : 330 = 10 : \frac{220}{3}$).

50th Sûtra.

(The *sûtra* is lost, but can be partially restored from the solution, and may be thus translated :—"The sum of the additive and subtractive numbers is divided by an assumed number; the quotient, lessened by the same number and halved, is squared and added to the subtractive number.")

Example.

Which number added to five is a square, that (same) number lessened by seven is a square. Which number is that? This is the question.

Statement :—$x + 5 = x^2$, and $x - 7 = x^2$.

Solution :—"The sum of the additive and subtractive numbers" is 12; the half of it is 6; lessened by two is 4; its half is 2; its square is 4. "And is added to the subtractive number;" the subtractive number is 7; added to it, it becomes 11 (i. e. $4 + 7$). This is that (required) number.

Its proof is this: $11 + 5 =$ square of 4 (*i. e.* 16); and $11 - 7 -$ square of 2 (*i. e.* 4).

(The next *sûtra* is only a fragment, and I omit it).

53rd Sûtra.

(Having found) the two fractions (indicative) of the daily earnings, divide by their difference what is given towards (producing) equal possessions. The quotient, being doubled, is the time (in which their possessions become equal).[19]

First Example.

Let one hired Paṇḍit earn five in three days; another learned man earns six in five days. The first gives seven to the second from his earnings. Say, in what time, after having given it, their possessions become equal?

Statement :—No. I, $\frac{5}{3} =$ earnings of 1 day; No. II, $\frac{6}{5} =$ earnings of 1 day; gift 7.

Solution : "The difference of the daily earnings; the two fractions; their difference;" (here the daily earnings are $\frac{5}{3}$ and $\frac{6}{5}$; their difference is $\frac{7}{15}$; the gift is 7; divided by the difference of the daily earnings $\frac{7}{15}$, the result is 15; being doubled, it is 30; this is the time), in which their possessions become equal.

Proof may be made by the rule of three :— $3 : 5 = 30 : 50$, and $5 : 6 = 30 : 36$; "the first gives seven to the second" 7, remainder 43; hence 43 and 43 are their equal possessions.

Second Example.

Two Râjpûts are the servants of a king. The wages of one (of them) per day are two and one-sixth, of the other one and one-half. The first gives to the second ten *dînâras.* Calculate and tell me quickly, in what time there will be equality (in their possessions)?

Statement :—daily wages $\frac{13}{6}$ and $\frac{3}{2}$; gift 10.

Solution :—"and difference of the daily earnings;" here (the daily earnings are $\frac{13}{6}$ and $\frac{3}{2}$: their difference is $\frac{2}{3}$; the gift is 10; divided by the difference of the daily earnings $\frac{2}{3}$, the result is 15; being doubled, it is 30. This is the time, in which their possessions become equal).

Proof by the rule of three :—$1 : \frac{13}{6} = 30 : 65$; and $1 : \frac{3}{2} = 30 : 45$. The first gives 10 to the second; hence 55 and 55 are their equal possessions.

(The following examples form a connected set. The *sûtras* to which they belong are very imperfectly preserved, nor is there any indication left, how they were numbered. The examples also exist in a too fragmentary state to allow of any translation: but it is possible to restore their purport from what is left of the solution.

The *sûtra* belonging to the following example is lost. The example itself may be reconstructed thus :—)

The second gives twice as much as the first, the third three times as much as the first, the fourth four times as much as the first. The total gift of the four persons is two hundred.

[19] The above is undoubtedly the meaning of the rule, though the exact construction of the text is not quite clear to me. Literally the words appear to be "The two fractions of the daily earnings cause their difference to divide, so that (*tat-yat*) the quotient, being doubled, is the time, that which is given towards equal possessions." *Tadviśêsham* and *dattâ* are the two accusatives governed by the causal verb *vibhâjayêt*.

Tell me now, how much was given by the first, and what is the amount of each gift.

Statement:—A gives x, B 2, C 3, D 4. Total 200.

Solution:—Having filled up the empty place (or x) with one, (we obtain) 1, 2, 3, 4 (as the several rates); by the application of the sum of the rates we obtain 20, 40, 60, 80, and thence the total 200.

This is the statement of the proof:—

Init. term 20, increment 20, period 4.

By the *rûpôṇa* method the total is found to be 200.

Sûtra.

(Only the first portion of this *sûtra* is preserved; *viz.* "put into the empty place the number 1 representing the desired quantity, and then make up the series of items." The purport of this rule will be understood from the following examples).

First Example.

(Its purport is:—B gives 2 times as much as A, C gives 3 times as much as B, D gives 4 times as much as C. Their total gift is 132. What is the gift of A?)

Statement:—A gives x, B 2, C 3, D 4. Total 132.

Solution:—"Put 1 in the place of x; then form the series of items" 1, 2, 3×2, 4×6, multiplying these several rates, 1, 2, 6, 24, their total is 33; with it divide the given total, thus $\frac{132}{33}$; the resulting item is 4, and this is the gift of A. Hence the series of gifts is as follows:—4, 8, 24, 96, and the total gift is 132. This is calculated from the series of items, and hence the total of the items is one hundred and thirty-two.

(Here follows what appears to be intended as a modification of the same *sûtra*, since it is not specialised as a separate *sûtra*. What remains of it, runs thus:—"the number 1 is put into the empty place, and then (the items) are successively multiplied." The purport of the rule will be again understood from the example.)

Second Example.

(Its purport is:—B possesses 2 times as much as A; C has 3 times as much as A and B together; D has 4 times as much as A, B and C together. Their total possessions are 300. What is the possession of A?)

Statement:—A has x, B 2, C 3×3, D 4×12. Total 300.

Solution:—"the desired quantity is put in the empty place;" the desired quantity is 1; this is placed as the first number; then the successive multiplications are made, 1, 2, 9, 48. Their addition gives the sum of the rates 60; with this the given total is divided, thus $\frac{300}{60}$; the result is 5, and this is the possession of A. With this by multiplication the several rates are obtained, thus 5, 10, 45, 240. Thence the total of the items is calculated to be 300.

(Next follows the fragment of a third example which I omit. After this must have followed a third modification of the same *sûtra*, which is lost; but the first portion of it, as quoted in the examples, must have run thus:—

Śûnyasthânê rûpaṁ datvâ, yutaṁ chaiva guṇaṁ tataḥ ।

i.e., "having put the number one in the empty place, the (needful) additions and multiplications are then made.")

Fourth Example.

(Its purport is:—A possesses something and $1\frac{1}{2}$ in addition; B has 2 times as much as A and $2\frac{1}{2}$ in addition; C has 3 times as much as B and $3\frac{1}{2}$ in addition; D has 4 times as much as C and $4\frac{1}{2}$ in addition. Their total possessions are one hundred and forty-four and one half. What is the possession of A?).

Statement:—A has $x + 1\frac{1}{2}$, B $2 + 2\frac{1}{2}$, C $3 + 3\frac{1}{2}$, D $4 + 4\frac{1}{2}$. Total $144\frac{1}{2}$.

Solution:—"Having put one in the empty place," thus $1 + 1\frac{1}{2}$: "the several additions and multiplications are then made:" in making the additions and multiplications, let the proper order of calculation be observed, (hence by addition) $\frac{5}{2}$; next comes multiplication; (here) multiply numerator with numerator and denominator with denominator, $\frac{10}{2}$ (*i.e.* $\frac{2}{1} \times \frac{5}{2}$); two and one half are now added, thus $\frac{15}{2}$: now comes the multiplication with the third number, or three (is multiplied) with seven and one half (*i.e.* $\frac{15}{2} = 7\frac{1}{2}$), thus $\frac{45}{2}$; three and one half are now added, thus $\frac{52}{2}$; now multiply the number four with twenty-six (*i.e.* $\frac{52}{2} = 26$); the result is $\frac{208}{2}$; four and one half are now added, thus $\frac{217}{2}$. The total of these rates is $\frac{259}{2}$ which is the given total of the possessions. All the rest remains the same; (*i.e.* dividing the given total $\frac{289}{2}$ by the sum of the rates $\frac{289}{2}$ we obtain 1 as the value of x, hence the possessions of A, B, C, D are respectively $\frac{5}{2}$, $\frac{15}{2}$, $\frac{52}{2}$ and $\frac{217}{2}$, the same as the rates mentioned above).

Fifth Example.

(Its purport is:—A gives $\frac{3}{2}$ *plus* a certain amount; B gives $\frac{5}{2}$ *plus* 2 times as much as A; C gives $\frac{7}{2}$ *plus* 3 times as much as A and B; D gives $\frac{9}{2}$ *plus* 4 times as much as A, B and C. The total of their gifts is 222. What was the gift of A?).

Statement:—A gives $x + \frac{3}{2}$, B $2 + \frac{5}{2}$, C $3 + \frac{7}{2}$, D $4 + \frac{9}{2}$; the joint gift is 222.

Solution:—"Having put the number one in the empty place," 1 (for x), the additions and multiplications are made in their proper order. The result is the following series of rates: $\frac{5}{2}$, $\frac{15}{2}$, $\frac{67}{2}$, $\frac{357}{2}$; the given total is 222. The addition of the rates yields 222, which is the same as the given total 222. This practically finishes the solution.

(Next follows the fragment of the sixth example, which I again omit).

Seventh Example.

(Its purport is:—A has $1\frac{1}{2}$ *plus* a certain amount; B has $2\frac{1}{2}$ less than 2 times A; C has $3\frac{1}{2}$ less than 3 times A; D has $4\frac{1}{2}$ less than 4 times A. Their total possessions are $\frac{29}{2}$. What is the possession of A?)

(The statement is wanting).

Solution:—"Having put the number one in the empty place," the addition is made $\frac{5}{2}$; twice the rate of A less five halves is $\frac{5}{2}$; three times the rate of A, less seven halves, is $\frac{8}{2}$; four times the rate of A, less nine halves, is $\frac{11}{2}$. The series of these rates is as follows: $\frac{5}{2}$, $\frac{5}{2}$, $\frac{8}{2}$, $\frac{11}{2}$. The given total is $\frac{29}{2}$. The sum of the rates is $\frac{29}{2}$. Dividing the one by the other, $\frac{29 \cdot 2}{2 \cdot 29}$, we obtain 1. Multiplying by this, the same amount is obtained (as the gift of A; *viz.* $\frac{5}{2}$) The same is the case with the negative quantities, (*i.e.* B $1 \times [(2 \times \frac{5}{2}) - \frac{5}{2}] = \frac{5}{2}$; similarly C $\frac{8}{2}$, D $\frac{11}{2}$).

NOTES.

1. In the text, the italicised words are conjecturally restored portions. The dots signify the syllables (*akshara*) which are wanting in the manuscript, the number of the dots corresponding to the number of missing syllables. The serpentine lines indicate the fact of lines being lost at the top and bottom of the leaves of the manuscript. In the translation the bracketed portions supply lost portions of the manuscript. The latter can, to a great extent, be restored by a comparison of the several examples. Occasionally words are added in brackets to facilitate the understanding of the passage.

2. **Sûtra 18.** Problems on progression. Two persons advance from the same point At starting *B* has the advantage over *A*; but afterwards *A* advances at a quicker rate than *B*. Question:—when will they have made an equal distance? In other words, that period of the two progressions is to be found where their sums coincide. The first example is taken from the case of two persons travelling. *B* makes 3 miles on the first day against 2 miles of *A*; but *A* makes 3 miles more on each succeeding day against *B*'s 2 miles. The result is that at the end of the third day they meet, after each has travelled 15 miles For *A* travels $2 + (2 + 3) + (2 + 3 + 3) = 15$ miles, and *B* $3 + (3 + 2) + (3 + 2 + 2) = 15$ miles. The second example is taken from the case of two traders. At starting *B* has the advantage of possessing 10 *dinâras* against the 5 of *A*; but in the sequel *A* gains 6 *dinâras* more on each day against the 3 of *B*. The result is that after $4\frac{1}{3}$ days, they possess an equal amount of *dinâras*, viz. 65.

3. **Sûtra 27.** Problems on averages (*samabhâgatâ*). Certain quantities of gold suffer loss at different rates. Question:—what is the average loss of the whole? The first problem is very concisely expressed; the question is understood; some words, like *kutô gatâ*, must be supplied to *samabhâgatâṁ*. The reading *rahitâ*, however, is not certain.

4. Brahmagupta's version of the **forty-ninth sûtra,** referred to above. (MS., No. I, B, 6. Library, As. Soc. Beng., p. 85) is as follows:—

Idânîm I

Yô râśir ishṭônô vargô bhavati, sô ch'ânyêshtayutô varga êva bhavati II

Tat-karaṇa-sûtram I

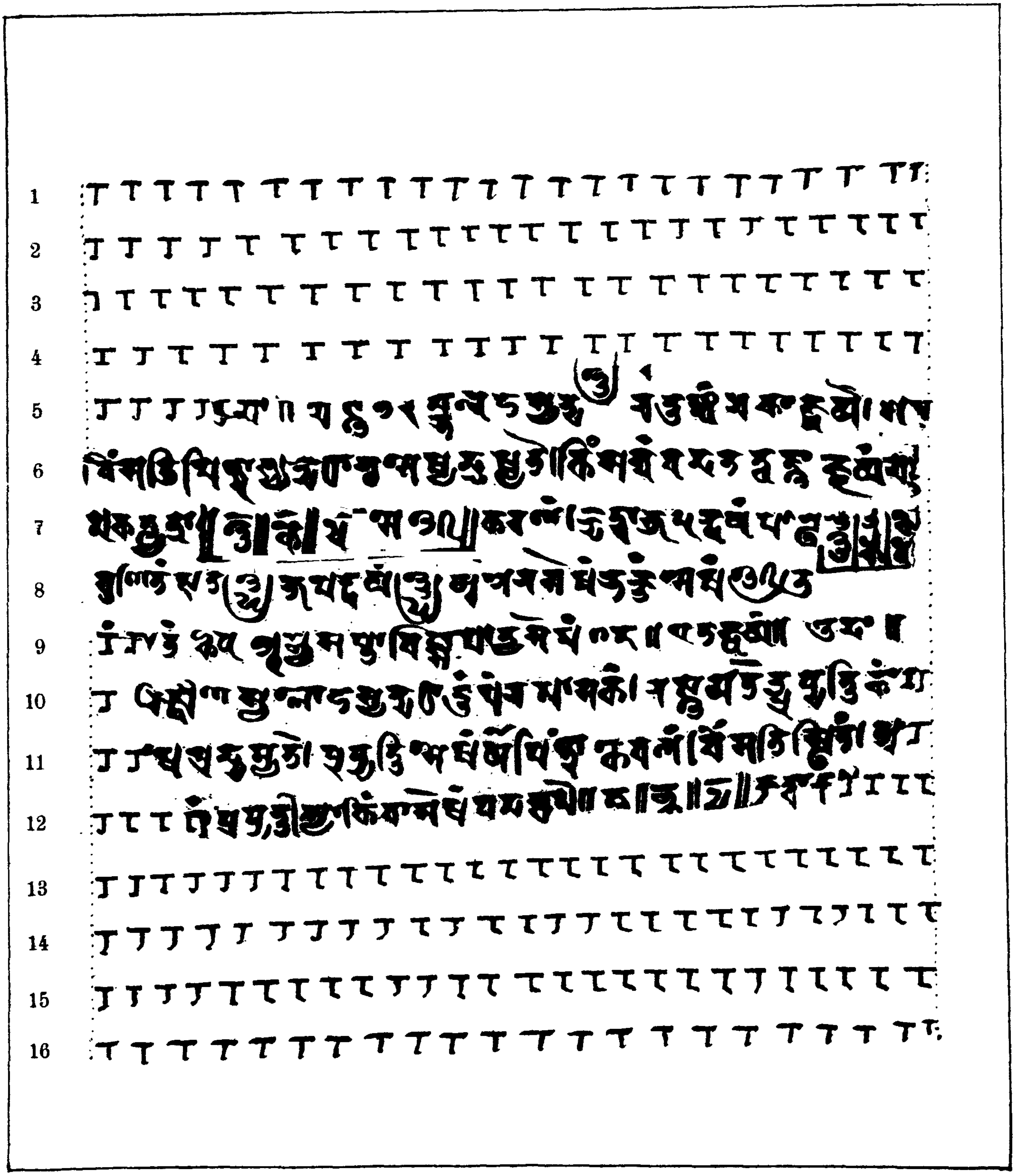
Yair ûnô yaiś cha yutô rûpair vargas tadaikyam ishṭa-hṛitam I

Ishṭônaṁ tad-dala-kṛitir ûnâbhyadhikâ bhavati râśiḥ II

FACSIMILE OF A LEAF OF THE BAKHSHÂLÎ MANUSCRIPT.

Containing a portion of Sûtra 25.

Plate I.



Size 8¼″ by 7″.

Obverse restored.

The *karaṇa-sûtra* is translated by Colebrooke (*Indian Algebra*, p. 371) thus :—"the sum of the numbers, the addition and subtraction of which makes the quantity a square, being divided by an arbitrarily assumed number (*ishṭa*), has that assumed number taken from the quotient : the square of half the remainder, with the subtractive number added to it, is the quantity (sought)." The *sûtra* is followed by a commentary and an example, which differs entirely from that given in the Bakhshâlî MS. It will also be noticed, that Brahmagupta's *sûtra* is in the *âryâ* measure, while the fragments of the *sûtra* in the Bakhshâlî MS., as restored from the solution, are in the *ślôka* measure.

5. **Unnumbered sûtras.** Problems on distribution or partition. It may be noted that these examples afford an illustration of what has been before remarked regarding the nature and use of the dot. It will be noticed that the dot • is called *śûnya* or 'the empty place :' and as the first step of the process of solution the direction is given 'to fill up the 'empty place with the number 1,' the latter being arbitrarily assumed to represent the unknown quantity of which the value is sought (the *ichchhâ* or *kâmika*).—The meaning of the example of the first *sûtra* is :—if A gives 1, B gives $2 \times 1 = 2$, C $3 \times 1 = 3$, D $4 \times 1 = 4$. The sum of the rates is $1 + 2 + 3 + 4 = 10$; dividing the given total 200 by 10, we obtain 20, as the gift of A. Hence the gifts of B, C, D are 40, 60, 80 respectively. The rule of the *rûpôna* method is not preserved in the Bakhshâlî MS. It is given, however, by Brahmagupta in the section of his Arithmetic on Progression. In Colebrooke's translation of Brahmagupta's work it is numbered 17 (on p. 290), and runs as follows :—"The period less one, multiplied by the common difference, being added to the first term, is the amount of the last. Half the sum of the last and first terms is the mean amount : which multiplied by the period, is the sum of the whole." Applying this rule to the present example we have :—the period 4 less 1 is 3 ; multiplied by the increment 20, it is 60 ; added to the initial term 20, it is 80. The sum of 80 and 20 is 100 ; half that sum is 50 ; and this multiplied by the period 4, yields the total 200. In the original Sanskrit (MS., No. I, B, 6, Library, As. Soc. Beng., p. 86) of Brahmagupta, the rule runs thus :—

Padâm êkahînam uttaraguṇitaṁ saṁyuktam âdinântyadhanaṁ ı
Âdiyut antyadhanârdhaṁ madhyadhanaṁ padaguṇitaṁ phalaṁ ıı

It will be noticed that this is in the *âryâ* measure, and that it is quite differently worded from the same rule in the Bakhshâlî MS., which commences with the word *rûpôna*, and which must have been in the *ślôka* measure. This confirms a remark previously made regarding the relation of the Bakhshâlî MS. to Brahmagupta.—In the fifth example of the second *sûtra* the rates are obtained thus: A gives $1 + \frac{3}{2} = \frac{5}{2}$; B $2 \times \frac{5}{2} + \frac{5}{2} = \frac{15}{2}$; C $3 \times (\frac{5}{2} + \frac{15}{2}) + \frac{7}{2} = \frac{67}{2}$; D $4(\frac{5}{2} + \frac{15}{2} + \frac{67}{2}) + \frac{9}{2} = \frac{357}{2}$. The sum of the rates is 222 ; dividing with this the given total 222, we obtain 1 as the value of x ; which practically finishes the problem : for multiplying each rate with 1, we obtain the same amounts $\frac{5}{2}$, $\frac{15}{2}$, $\frac{67}{2}$, $\frac{357}{2}$ for the several gifts of A, B, C and D.

6. The page figured on the accompanying plate reads as follows :—

1\. . . . . . udâ [ajñâta-r-ambhalôhasya tṛi-chatuḥ-paṁchakâ kshayê ı sapta =

2\. viṁśati piṇḍisya tṛidhânta-śêshya drishyatê ı kiṁ sarvaṁ vada tatvajña kshayaṁ cha

3\. ma katthyatâṁ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ śe 27 karaṇaṁ ı kṛitva rûpa-kshayaṁ pârtha $\left|\frac{2}{3}\middle|\frac{3}{4}\middle|\frac{4}{5}\right|$

4\. guṇitaṁ jâta $\left|\frac{2}{5}\right|$ rûpa-kshayaṁ $\left|\frac{3}{5}\right|$ anêna śêshaṁ bhaktaṁ śêshaṁ | 27 | bha =

5\. [ktaṁ] jâtaṁ 45 asya saptâviṁśa pâtya śêshaṁ 18 ıı êta kshayaṁ ıı udâ ı

6\. [pa]rikshîṇasya lôhasya tṛidhântâṁ paṁchamâshakaṁ ı na jñâyatê [ta]t-pravṛittika

7\. [na śê]sha pradṛiśyatê ı pravṛitti-śêshaṁ yô piṇḍaṁ kêvalaṁ viṁśati sthitaṁ ı â =

8\. darśyatâṁ pravṛittî syâ kiṁ vâ śêshaṁ vadasva mê $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ kṛitvâ [rûpa =]

The lithographed plate, unfortunately, is not quite perfect. The transcript has been made from the original. *Ambha-lôha* I take to be the Sanskrit *abhra-rôha* 'lapis lazuli' (cf. Pâli *ambhô* 'a pebble'). For *paṁchamâśakaṁ* read *paṁchamâṁśakaṁ*. The purport of the first example is: "of an unknown quantity (*piṇḍa*) of lapis lazuli, on deducting the loss (in cutting), there remain $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ in three instalments (*tridhâ-anta*); the sum of the remainders of the three instalments is 27. What was the total, and what is the loss?" Solution: "Subtracting from 1 severally $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, we get $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$; these multiplied with one another are $\frac{2}{5}$; subtracting this from 1, we get $\frac{3}{5}$; the total remainder 27, being divided by this, we get 45; deducting from this the total remainder 27, we get 18 as the loss." Proof: the total is 45; at the first time of cutting, $\frac{1}{3}$ or 15 is got as cut stones; hence the loss (or what is cut away) is 30; the latter is cut once more, and $\frac{1}{4}$ or $7\frac{1}{2}$ is got as cut stones, the loss being $22\frac{1}{2}$; this is cut a third time, and $\frac{1}{5}$ or $4\frac{1}{2}$ is now got as cut stones, the final loss being 18. The produce of the three instalments of cutting, accordingly, is $15 + 7\frac{1}{2} + 4\frac{1}{2}$ or 27.—The second example is similar; only that here, besides the original (*pravṛitti*) total (50), the total produce (*śêsha* or what remains after deducting the several losses) is to be found (30), instead of the final remainder (*pravṛitti-śêsha*) which is given as 20. The solution (and proof) may be made exactly as in the case of the first example.

SOMALI AS A WRITTEN LANGUAGE.

No. III.[1]

BY CAPTAIN J. S. KING, Bo.S.C.

COLLOQUIAL SENTENCES.

English	Somâli
18.—How old are you?	ادگ اِتمس جربتهی or ادگ اِمساد جرتی
19.—Will you sell this?	ادگ وه ما ابذیس
20.—Yes: I will sell it.	ها و اِبذی
21.—Will you buy this?	ادگ وه ما اِبسذیس
22.—I will buy it.	و اِبسذی
23.—I shall beat you.	انگ و کگ دفن
24.—Hold my horse.	فرسکذیگی قبو
25.—I will hold it.	و قبذی
26.—What have you brought?	مهاد کیذتی
27.—Bring me a good spear.	ورن وناقسن اِیکین
28.—I want a mat.	ذرمان دونی
29.—Do you know what he says?	و هو لیهیهی مقن
30.—Is this knife yours?	مندید متادیب
31.—Yes: this is mine.	ها وه اباليده
32.—Is much coffee produced in your country?	مغالداد بن بدن مليدهی
33.—What is the charge for a camel-load?	اورك قاد کیسي و اِتمس
34.—Is any fresh water procurable here?	میش بِیو معن مليدهي
35.—How far is the town from the shore?	مغالد هیبت اِتمس جرت
36.—I saw you to-day in the bazar.	مانت سوگذي یان کو ارقی
37.—What were you doing there?	میش مهاد کسمیذیسی
38.—I was buying some food.	وه هان ارتو اِبسذی
39.—I shall come to your house to-day.	مانت اغلکاگي یان اِمذی
40.—I want some bread and salt.	اکبس آیو ارسبوه یان دونی

[1] *Erratum in No. II. Somâli.* The last sentence in the left-hand column on page 285, Vol. XVI., should be written as follows:—"By this method the student is saved the trouble of wading through grammatical rules (which, without practice in their use, would probably not convey much information to his mind), and his attention is drawn only to those points of grammar which arise in the sentences."